YOUNG WOMEN IN MATHEMATICAL PHYSICS

University of Bonn, 24-26 September 2018

Poster Session

ROBERTA BIANCHINI, *On a vector-BGK approximation of fluid-dynamics equations*

We consider a singular semilinear hyperbolic approximation to hydrodynamic equations in 2D, inspired by the kinetic theory. We show the convergence of the vector-BGK model to the incompressible Navier-Stokes equations under the diffusive scaling. Our strategy is based on the use of local in time Sobolev estimates, combined with the relative entropy and the interpolation properties of the Sobolev spaces.

MARINA FERREIRA, *A time-stepping algorithm for ballistic aggregation*

This poster deals with the problem of simulating the dynamics of large and densely packed particle systems subject to ballistic aggregation. Two different paradigms are typically used: event-driven (ED) and time-stepping (TS) algorithms. Despite being more accurate, ED algorithms become computationally very expensive as the number of particles increases. Such situations require the use of an alternative approach, such as TS algorithms. In these methods, the contact events occurring during a small time-interval are grouped and solved simultaneously. In this work we explore a new TS approach for the case of ballistic aggregation and we compare it to the ED one in terms of computational time. This approach is based on a minimization problem with non-overlapping constraints that is solved with the recently proposed damped Arrow-Hurwicz algorithm. Specifically, we obtain four TS algorithms from combining smooth or non-smooth constraints with rigid or non-rigid aggregates. Numerical results obtained in the case of a bounded domain and spherical particles show that when the number of particles becomes large or the size of the domain becomes small the TS methods perform faster than the ED. The threshold at which this switch occurs is identified. In particular, for N=1 000 000 particles, the ED takes almost four days, while the TS takes only a few hours.

CHIARA FRANCESCHINI, *An algebraic approach to Markov duality.*

Duality in the context of stochastic processes is a remarkable tool to deal with interacting particles systems and diffusions. Once a duality relation is available, several applications applies but it is also interesting to understand the mathematical structure behind duality.

SUSANNE HILGER, *Statistical Mechanics of Gradient Interface Models*

A discrete gradient model for interfaces is studied. The interaction potential is a non-convex perturbation of the quadratic gradient potential. As in the non-perturbed and convex case we can prove that the field scales to a Gaussian Free Field in the continuum limit and the covariance decays algebraically in the thermodynamic limit. The proof is an extension of the renormalisation group method to observables.

MELISSA MEINERT, *Sobolev spaces and calculus of variations on fractals.*

We consider Sobolev spaces on metric measure spaces that carry a strongly local regular Dirichlet form. Our aim is to generalize some basic results from the calculus of variations, such as the existence of minimizers for convex functionals, with the help of these Sobolev spaces. This applies to a few non-classical situations such as degenerate diffusions, superpositions of diffusions and diffusions on fractals or on products of fractals. Based on joint work with Michael Hinz and Dorina Koch.

Anna Muranova, *Conservation of complex power in electric networks.*

In [2] and [5] the electric networks with resistors are being studied using weighted graphs. Also, the concept of the effective resistance of the network is introduced. It is shown that due to the Kirchoff's law the problem of calculating the effective resistance is related to the Laplace operator and the Dirichlet problem on graphs.

If one consider the electric network with passive elements (inductors, capacitors and resistors) and alternating current, then for calculating the effective impedance (analogue of effective resistance) complex-weighted graphs can be used.

We introduce the concept of a *complex-weigted graph* and *Laplace operator* on it. Then we introduce the concept of *network* and *Dirichlet problem* related with it. We define an *effective impedance* of the finite network and prove that it is well-defined and satisfies some basic physical properties.

The main result is the *law of conservation of complex power*. As a corollary of this law, we obtained that if the network consists of just passive elements, then its effective impedance has non-negative real part.

References:

[1] Charles A. Desoer and Ernest S. Kuh. *Basic circuit theory.* 1969.

[2] Peter G. Doyle and J. Laurie Snell. *Random walks and electric networks.* 2006.

[3] Alexander Grigoryan. *Analysis on Graphs. Lecture notes.* Bielefeld Uni- versity, WS 2011/12.

[4] R. P. Feynman. *The Feynman lectures on physics, Volume 2: Mainly Electromagnetism and Matter.* 1964.

[5] David A. Levin, Yuval Peres, Elizabeth L. Wilmer. *Markov Chains and Mixing Times.* 2009.

[6] Arieh L. Shenkman. *Circuit analysis for power engineering handbook.* 1998

Lisa Onkes, *Singularity Formation for Dispersive Waves*

Parts of a solution to a *dispersive PDE* with different spatial frequencies travel with different velocities: The solution "disperses" over time since *simple wave solutions* $(t,x) \mapsto Ae^{i(kx-\omega t)}$ average rather than concentrate. Therefore, the amplitude of a solution tends to decrease as time evolves. I am interested in equations where a focusing nonlinearity counters the dispersive behavior, and, due to this, solutions might blow up. A well-known prototype of this setting is the generalized Korteweg-de-Vries equation

$$u_t + u_{xxx} = -(|u|^{p-1}u)_x, \qquad \text{(gKdV)}$$

where one can observe a competition between the equation $u_t + u_{xxx} = 0$ (which smoothes u in space as time evolves) and the equation $u_t = -(|u|^{p-1}u)_x$ that might lead to blow-up in finite time.

Specifically, we are studying the (gKdV) with supercritical nonlinearities ($p > 5$) for which physical experiments and numerical simulations suggest so-called *selfsimilar blow-up solutions*. We give a rigorous proof of the existence and asymptotic stability of a selfsimilar blow-up solution for all supercritical nonlinearities.

Chinar Rana, *Effects of nonlinear adsorption on miscible viscous fingering in a homogeneous porous medium*

Transport phenomenon is of fundamental as well as practical importance in a wide spectrum of problems of different length and time scales, viz., enhanced oil recovery, carbon capture and storage, contaminant transport in subsurface aquifers, and chromatographic separation. These transport processes in porous media feature different hydrodynamic instabilities. Viscous fingers (VF) appear due to such an instability mechanism attributed to the mobility mismatch between the invading and defending fluids–a less mobile defending fluid allows the finger-like intrusion of a more mobile invading fluid. In this talk, we will discuss the interactions of viscous fingers with shock layer or rarefaction. Our approach is to combine two well-studied, independent settings–viscous fingering dynamics in miscible fluids and shock layer (rarefaction) wave fronts–to investigate how these two nonlinear dynamics impact one another. Our analytical approach treats a solvent containing a finite slice of an inert solute that (a) is adsorbed on the porous matrix following a Langmuir or anti-Langmuir adsorption isotherm and (b) influences the dynamic viscosity of the solution. The difference between the cases with Langmuir and anti-Langmuir arises in creation/annihilation of the nonlinear waves as well as viscous fingering. Viscous fingers formed at a viscously unstable rarefaction interface propagate through the finite sample to pre-empt the shock layer at the viscously stable front.

Cinzia Soresina, *Cross-diffusion predato-prey models arising by time-scale arguments.*

Starting from microscopic models incorporating the dynamics of handling and searching predators, or active and hidden prey, we obtain reaction cross-diffusion systems of predator-prey type involving classical functional responses, by the exploitation of different time-scales. We also provide a study of the Turing instability domain of the obtained equations and (in the case of the Beddington-DeAngelis functional response) compare it to the same instability domain when the cross-diffusion is replaced by a standard diffusion.