Gradient theories for plasticity as the Γ-limit of a nonlinear dislocation energy

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Since the motion of dislocations is regarded as the main cause of plastic deformation, a large literature is focused on the problem of deriving plasticity models from more fundamental dislocation models. The starting point of our derivation is a semi-discrete dislocation model. The main novelty of our approach is that we consider a nonlinear dislocation energy, whereas most mathematical and engineering papers treat only a quadratic dislocation energy. Our choice of a nonlinear stress-strain relation guarantees that the dislocation strain energy is well defined also in the vicinity of the dislocations, eliminating the need of the cut-off radius that is typical of the linear theories.

We consider both the case of a finite number of dislocations (dilute case) and the critical case where the number of dislocations goes to infinity. We study the asymptotic behaviour of our nonlinear dislocation energy as the length of the Burgers vector tends to zero via Gamma-convergence. In the limit we obtain strain-gradient models for plasticity of the same form as the limit energies derived from a quadratic dislocation energy. Our result, however, is obtained by starting from a more physical model.

References

[1] Scardia, L. and Zeppieri, C.: *Line-tension model for plasticity as the* Γ*-limit of a nonlinear dislocation energy*, SIAM J. Math. Anal., to appear.