Young Women in PDEs

Bonn, 10–12 May, 2012

Conference program

Institute for Applied Mathematics
University of Bonn

universitätbonn iam

Bonn International Graduate
School in Mathematics

Thursday, 10 May

09:00 - 09:15 *Welcome*

09:15 - 11:00 Nina Uraltseva, *"Regularity in free boundary problems"*

11:00 - 11:30 *Coffee break*

11:30 - 12:10 Betul Orcan-Ekmekci, *"About the Geometry and Regularity of Largest Subsolutions for a Free Boundary Problem in* $\mathbf{R}^2$*: Elliptic Case"*

12:10 - 12:50 Agnese Di Castro, *"Measure data problems, lower order terms and interpolation effects"*

12:50 - 14:30 *Lunch break*

14:30 - 15:10 Melanie Rupflin, *"Flowing maps to minimal surfaces"*

15:10 - 15:50 Nasrin Arab, *"Nonlinear stability of stationary solution for Surface Diffusion with triple junction"*

15:50 - 16:20 *Coffee break*

16:20 - 17:00 Andrea Fernández, *"Single- and two-scales sharp-interface models for concrete"*

17:00 - 17:40 Olena Vaneeva, *"Applications of equivalence and symmetry point transformation in studying PDEs"*

18:00 - 19:00 Poster session

19:30 - *Reception*

Friday, 11 May

09:15 - 11:00 Maria Westdickenberg, *"Dynamic and Stochastic Metastability: Problems at the boundary of applied analysis, PDE, and probability theory"*

11:00 - 11:30 *Coffee break*

11:30 - 12:10 Chiara Saffirio, *"An existence result for the 3d repulsive plasma-charge model"*

12:10 - 12:50 Claudia Hecht, *"Analysis for optimal control of the elastic Allen-Cahn system"*

12:50 - 14:30 *Lunch break*

14:30 - 15:10 Annalisa Massaccesi, *"Currents with coefficients in a group and the Steiner Problem"*

15:10 - 15:50 Kim Hang Le Nguyen, *"The periodic unfolding method for a class of imperfect transmission problems"*

15:50 - 16:20 *Coffee break*

16:20 - 17:00 Elisabetta Chiodaroli, *"Non-uniqueness results for entropy solutions to the compressible Euler system"*

17:00 - 17:40 Jelena Aleksić, *"On the compactness of solutions to multidimensional conservation law with discontinuous flux"*

18:00 - 19:00 Poster session

19:30 - *Social dinner*

Saturday, 12 May

09:15 - 11:00 Adriana Garroni, *"Multiscale variational analysis of dislocations"*

11:00 - 11:30 *Coffee break*

11:30 - 12:10 Lucia De Luca, *"Γ-convergence analysis of systems of edge dislocations"*

12:10 - 12:50 Lucia Scardia, *"Gradient theories for plasticity as the Γ-limit of a nonlinear dislocation energy"*

12:50 - 13:30 Elisa Davoli, *"A quasistatic evolution model for perfectly plastic plates derived by Γ-convergence"*

13:30 - 14:30 *Closing & Snack*

Abstracts

Main lectures

Multiscale variational analysis of dislocations

Adriana Garroni

The main mechanism for crystal plasticity is the formation and motion of a special class of defects, the dislocations. These are topological defects in the crystalline structure that can be identify with lines on which energy concentrates. In recent years there has been a considerable effort for the mathematical derivation of models that describe these objects at different scales (from an energetic and a dynamical point of view). The description of the problem is extremely complex in its generality. The lecture will be devoted to present some variational models for dislocations that can be obtained, by Γ-convergence, starting from discrete or semi-discrete energies. I will discuss the relation between these models and other classical models in applied mathematics (as Ginzburg-Landau models for vortices or non-local Cahn-Hilliard energies for phase transitions).

Regularity in free boundary problems

Nina Uraltseva

Free boundary problems cover a wide class of nonlinear problems for partial differential equations and have applications in science, industry, medicine and economics. The problems with free boundaries describe processes with possible jump like changes. Moreover, it is *a priori* unknown at what time and where such a jump happens. Typical here are the processes with phase transitions in the medium, the phenomenon of hysteresis, the biological processes related to the population dynamics. Some models of financial markets can be also considered as free boundary problems. In this talk we discuss the methods, developed in the last decades, for investigation the problems with free boundaries. These methods include scalings, various monotonicity formulas, study of global solutions, applications of ideas from the geometric measure theory and from the theory of nonlinear PDEs.

Dynamic and Stochastic Metastability: Problems at the boundary of applied analysis, PDE, and probability theory

Maria Westdickenberg

In this talk we describe two distinct and intriguing phenomena in applied analysis: dynamic metastability and stochastic metastability. Dynamic metastability of a dynamical system refers to the phenomenon in which the system relaxes quickly to a so-called dynamically metastable state, a lower-dimensional subspace of phase space that is not stable, but such that the evolution within this subspace is very slow. After a long time in or near the metastable state, the system may undergo drastic change, moving to a part of phase space that is far away. Hence, from both a physical and mathematical point of view, it is important to distinguish metastable states. Moreover, we need new analytical and computational tools to understand them. We describe recent progress.

Stochastic metastability, on the other hand, refers to noise-induced exit from a stable state. Here the underlying deterministic dynamics is relaxation to a stable state, but the noise occasionally induces a so-called rare event in which the system goes against the deterministic ow. What is the probability of such an event? Which kinds of random paths are most likely to be observed? The theory of large deviations and its extensions provide a link between these questions from probability and tools from analysis.

Contributed talks

On the compactness of solutions to multidimensional conservation law with discontinuous flux

Jelena Aleksić and Darko Mitrovic

We consider the following Cauchy problem for multidimensional scalar conservation law

$$u_t + \operatorname{div} f(x,u) = 0, \qquad u(x,0) = u_0(x),$$

where $u = u(t,x)$, $x \in \mathbf{R}^d$, $t \in \mathbf{R}^+$ and $f = (f_1, ..., f_d) : \mathbf{R}^{d+1} \to \mathbf{R}^d$, $d \in \mathbf{N}$. For the initial data u_0 we assume that $u_0 \in (BV \cap L^\infty)(\mathbf{R}^d)$, $\quad a \le u_0(x,y) \le b, \quad x \in \mathbf{R}^d$. The flux function f has the following properties: $f_i(\cdot, \lambda) \in (BV \cap L^\infty)(\mathbf{R}^d)$, for every $\lambda \in \mathbf{R}$, $f_i(x, \cdot) \in Lip(\mathbf{R})$, for every $x \in \mathbf{R}^d$, $0 = f(x,b) = f(x,a)$, for every $x \in \mathbf{R}^d$.

We analyze a family of solutions to a regularization of the mentioned problem by smoothing flux function and initial data and involving the vanishing viscosity. We present a new genuine nonlinearity condition, weaker then in previous works on the subject, and prove strong L^1_{loc}-precompactness of mentioned family of solutions.

References

[1] J. Aleksic and D. Mitrovic: *On the compactness for two dimensional scalar conservation law with discontinuous flux*, Commun. Math. Sci. 7 (2009), no. 4, 963-971.

[2] E. Yu. Panov: *Existence and strong pre-compactness properties for entropy solutions of a first-order quasilinear equation with discontinuous flux*, Arch. Ration. Mech. Anal. 195 (2010), no. 2, 643-673.

Nonlinear stability of stationary solution for Surface Diffusion with triple junction

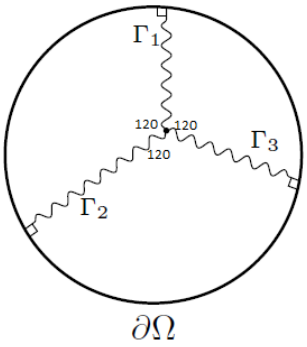
Nasrin Arab, Helmut Abels, and Harald Garcke

In the talk we will analyze the motion of a network of three curves in a ball having perpendicular intersection with the outer boundary and a common intersection at a triple junction with 120 degree angle conditions. We assume they move under Surface Diffusion

Flow namely with a speed proportional to the negative Laplace-Beltrami of the curvature of the curves. It is a nonlinear fourth-order parabolic equation with nonlinear boundary conditions. The flow decreases the total length of the curves and preserves the enclosed areas and in fact it has a H^{-1}-gradient flow structure.

We discuss the nonlinear stability of the stationary solution having the form of a Mercedes-Benz star using *the generalized principle of linearized stability*. We have shown that the stationary solution is normally stable and the linearized problem satisfied *Lopatinskii-Shapiro* and *normally ellipticity* condition which is needed for maximal regularity.

One of the difficulties which arises here is highly nonlocality in space due to the movement of triple junction. So in order to deal with this problem we use parabolic hölder settings. and we have proved the local existence.



A non-uniqueness result for entropy solutions to the compressible Euler system

Elisabetta Chiodaroli

The deceivingly simple-looking compressible Euler equations of gas dynamics have a long history of important contributions over more than two centuries. If we allow for discontinuous solutions, uniqueness and stability are lost. In order to restore such properties, further restrictions on weak solutions have been proposed in the form of entropy inequalities. In this talk we will discuss a counterexample to the well-posedness of entropy solutions to the multi-dimensional compressible Euler equations (see [1]): in our construction the entropy condition is not sufficient as a selection criteria for unique solutions. Our methods are inspired by a new analysis of the incompressible Euler equations recently carried out by De Lellis and Székelyhidi in [3]-[4] and based on a revisited "h-principle".

References

[1] E. Chiodaroli: *A counterexample to well-posedeness of entropy solutions to the compressible Euler system*, Preprint, 2011

[2] E. Chiodaroli and C. De Lellis: *Non-standard solutions of the p-system with Riemann data*, In preparation

[3] C. De Lellis and L.J. Székelyhidi: *The Euler equations as a differential inclusion*, Ann. Math., 170: 101-120, 2009

[4] C. De Lellis and L.J. Székelyhidi: *On admissibility criteria for weak solutions of the Euler equations*, Arch. Rational Mech. Anal., 195: 225-260, 2010

A quasistatic evolution model for perfectly plastic plates derived by Γ-convergence

Elisa Davoli and Maria Giovanna Mora

We consider an evolutionary dimension reduction problem for a thin plate whose elastic behaviour is linear and isotropic and whose plastic behaviour is governed by the Prandt-Reuss flow rule without hardening (perfect plasticity). A lower dimensional model is deduced by Γ-convergence. A convergence result for a sequence of 3D quasistatic evolutions to a reduced quasistatic evolution for the limit model is established.

References

[1] E. Davoli and M.G. Mora: *A quasistatic evolution model for perfectly plastic plates derived by Gamma-convergence*, in preparation.

Γ-convergence analysis of systems of edge dislocations

Lucia De Luca, Adriana Garroni, and Marcello Ponsiglione

Dislocations are the most common defects in crystals and their presence is considered the main mechanism of plastic deformations in metals.

We shall consider the elastic energy induced by systems of straight edge dislocations in the framework of linearized plane elasticity. In this context, dislocations can be introduced as point topological defects of the displacement gradient fields. Following the core radius approach, we will introduce a parameter $\epsilon > 0$ representing the lattice spacing of the crystal and we will compute the elastic energy stored outside the core region, namely the union of the discs with radius ϵ centered at each dislocation. We shall analyze in terms of Γ-convergence the asymptotic behavior of the elastic energy as $\epsilon \to 0$. We will focus

on the $|\log \epsilon|$ regime, showing a compactness result for sequence of configurations with logarithmic diverging energy. In particular, we will see as this analysis presents many similarities with the one of the Ginzburg-Landau functionals for the study of the vortices in superconductivity with specific difficulties due to the vectorial nature of the problem.

References

[1] L. De Luca, A. Garroni, and M. Ponsiglione: Γ-*convergence analysis of systems of edge dislocations: the self energy regime*, Submitted paper.

[2] E. Sandier and S. Serfaty: *Vortices in the Magnetic Ginzburg-Landau Model*, Birkhäuser, 2007.

Measure data problems, lower order terms and interpolation effects

Agnese Di Castro and Giampiero Palatucci

We deal with the solutions to nonlinear elliptic equations of the form

$$-\operatorname{div} a(x, Du) + g(x, u) = f,$$

with f being just a summable function, under standard growth conditions on g and a. We prove general local decay estimates for level sets of the gradient of solutions in turn implying very general estimates in rearrangement and non-rearrangement function spaces, up to Lorentz-Morrey spaces. The results obtained are in clear accordance with the classical Gagliardo-Nirenberg interpolation theory.

Single- and two-scales sharp-interface models for concrete

Andrea Fernández and Jonathan D. Evans

There is a need to design durable concrete structures. A fundamental understanding of the deterioration phenomena is therefore vital. Concrete carbonation is one of the most common ways in which reinforced concrete buildings can be damaged. In this talk I will present a one- dimensional model that predicts concrete carbonation. In the physically relevant limit of a fast bulk reaction, a matched asymptotic approach is used to derive sharp interface models that correspond to different scalings in the small parameter. The resulting models give one-phase and two-phase Stefan moving-boundary problems with interface conditions determined by a micro problem. This micro-macro moving boundary problem will be discussed in three conceptually different regimes for the diffusivities of the driving chemical species.

Analysis for optimal control of the elastic Allen-Cahn system

Claudia Hecht

In this talk we describe interface evolution with the help of a phase field approach, while taking elastic effects into account. This theory is used for example to model spinodal decomposition and coarsening in materials science.

Here we discuss the Allen-Cahn system describing the interface evolution, which is extended by some mechanic equations including the elastic effects. Altogether we get a second order semi-linear parabolic equation coupled to an elliptic system. Similar considerations already have been carried out for the related Cahn-Hilliard system. Moreover, we consider an optimal control problem, where the state equations are given by the elastic Allen-Cahn system. We control the evolution with a forcing-term on a part of the boundary. We start considering first fundamental mathematical questions with respect to this optimal control problem. Eventually, we obtain first order optimality conditions for this optimal control problem.

The periodic unfolding method for a class of imperfect transmission problems

Kim Hang Le Nguyen, Patrizia Donato, and Roland Tardieu

The periodic unfolding method was introduced by D. Cioranescu, A. Damlamian and G. Griso in [1] for studying the classical periodic homogenization in fixed domains and

more recently extended by them, together with P. Donato and R. Zaki, to periodically perforated domains in [2].

In our work, the method is adapted to two-component domains which are separated by a periodic interface. The first component is connected (like perforated domains) and the second one is a disconnected union of periodic sets (like the zone usually occupied by periodic holes). We introduce a second unfolding operator, acting on functions defined in the disconnected part, and study its main properties. Moreover, we study the relationship between the two operators and in particular the properties of their traces on the common boundaries.

The unfolding method is then applied to an elliptic problem with a jump of the solution on the interface, which is proportional to the flux and depends on a real parameter. We prove some homogenization and corrector results, which recover and complete those previously obtained in [3] by the first author and S. Monsurrò.
The results have been recently published in [4].

References

[1] D. Cioranescu, A. Damlamian, and G. Griso: *Periodic unfolding and homogenization* C. R. Acad. Sci., Paris, Ser. I, Math. 335, 99-104, 2002.

[2] D. Cioranescu, A. Damlamian, P. Donato, G. Griso, and R. Zaki: *The periodic unfolding method in domains with holes*, [Submitted]

[3] P. Donato and S. Monsurrò: *Homogenization of two heat conductors with an interfacial contact resistance*, Anal. Appl., Singap. 2, No. 3, 247-273, 2004.

[4] P. Donato, K.H. Le Nguyen, and R. Tardieu: *The periodic unfolding method for a class of imperfect transmission problems*, Journal of Mathematical Sciences, 176 (6), 891- 927, 2011.

Currents with coefficients in a group and the Steiner Problem

Annalisa Massaccesi and Andrea Marchese

The Steiner problem consists in finding the shortest connected set including some fixed points; we show how this problem could be solved as a mass-minimizing problem for 1-dimensional rectifiable currents with coefficients in a suitable group. Thanks to the representation adopted for these currents, we can exploit the calibration method and we can analyse some examples.

About the geometry and regularity of largest subsolutions for a free boundary problem in $\mathbf{R}^2$: elliptic case

Betul Orcan-Ekmekci

In this study, we present some geometric and regularity properties of the largest subsolution of a one-phase free boundary problem under a very general free boundary condition in $\mathbf{R}^2$. Moreover, we provide density bounds for the positivity set and its complement near the free boundary.

Free Boundary Problem (FBP) that we would like to analyze is the following: for a given bounded open domain $D \subseteq \mathbf{R}^2$ and $u : \mathbf{R}^2 \setminus D \mapsto [0, +\infty)$ is a continuous function which satisfies:

$$\triangle u = 0 \quad \text{in} \quad \Omega(u) \backslash D, \tag{1}$$

$$u = g(x) \quad \text{on} \quad \partial D, \tag{2}$$

$$u = 0,\ |\nabla u|^2 = f(x) \quad \text{on} \quad \partial\Omega(u), \tag{3}$$

where $\Omega(u) = \{x \in \mathbf{R}^2 | u(x) > 0\}$; $g(x)$ and $f(x)$ are positive continuous functions. For $f(x)$, there exist $\Lambda, \lambda > 0$ such that $0 < \lambda < f(x) < \Lambda$, for all $x \in \mathbf{R}^2$.

There is a wide range of physical models related to the above FBP, encompassing problems such as flame propagation and G-equations, capillary drops on a flat or inclined surface, phase transitions, and obstacle problems. There are previous results about the regularity of variational and weak solutions to these example FBPs. Most of these results require that the Free Boundary Condition (FBC) is at least Lipschitz and the media is periodic, we would like to extend these results both to viscosity solutions and to the random case since real life systems also require to work with these cases. In the random case, media can be heterogenous without any periodic setting, i.e. the FBC can be at most positive, bounded, and continuous in the space variable. In this study, we focus on regularity issues for a FBP related to these phenomena and we concentrate on the geometric description of the largest viscosity subsolution in two dimensions with weaker requirements on the data. We develop a regularity and non degeneracy theory for it's largest subsolution and give a nice geometric characterization of the free boundary. Motivated by the study of random media, we allow for the data to be highly oscillatory. Thus, we only require $f(x)$ to be positive, bounded, and measurable function. We used the continuity of $f(x)$ only to be able to obtain the continuous viscosity solutions. One can even weaken the continuity assumption on $f(x)$ by taking into account suitable viscosity solution definitions.

Flowing maps to minimal surfaces

Melanie Rupflin and Peter Topping

We explore the idea of defining a geometric evolution equation that deforms a given surface into a minimal surface, that is into a critical point of the area functional.

The flow we define has elements in common with two well known parabolic evolution equations; the mean curvature flow which is the negative gradient flow of the area and the harmonic map heat flow which is the negative gradient flow of the Dirichlet energy. Using a combination of these two ideas has the advantage that we can profit of the much better analytical properties of the energy and thus of the harmonic map heat flow while still getting an object in the limit $t \to \infty$ that is not only a critical point of the energy (i.e. harmonic) but indeed a critical point of the area (i.e. minimal).

We recover the classical results of Sacks-Uhlenbeck and Schoen-Yau about the existence of (branched) minimal immersions, originally proved by direct minimisation technique, with the (branched) minimal immersion now obtained as the limit of the flow starting from any admissible initial data.

An existence result for the 3d repulsive plasma-charge model

Chiara Saffirio, Laurent Desvillettes, and Evelyne Miot

We establish global existence of a solution to the three-dimensional Vlasov-Poisson system in presence of a point charge with repulsive interaction. The present analysis extends the analogous result for the three-dimensional Vlasov-Poisson system without charges in [1]. Moreover, we recover all additional regularity properties on the electric field which have been establishes in [1].

References

[1] P. L. Lions and B. Perthame: *Propagation of moments and regularity for the 3-dimensional Vlasov-Poisson system*, Invent. Math., 105: 415-430, 1991.

Gradient theories for plasticity as the Γ-limit of a nonlinear dislocation energy

Lucia Scardia and Caterina Ida Zeppieri

Since the motion of dislocations is regarded as the main cause of plastic deformation, a large literature is focused on the problem of deriving plasticity models from more fundamental dislocation models. The starting point of our derivation is a semi-discrete dislocation model. The main novelty of our approach is that we consider a nonlinear dislocation energy, whereas most mathematical and engineering papers treat only a quadratic dislocation energy. Our choice of a nonlinear stress-strain relation guarantees that the dislocation strain energy is well defined also in the vicinity of the dislocations, eliminating the need of the cut-off radius that is typical of the linear theories.

We consider both the case of a finite number of dislocations (dilute case) and the critical case where the number of dislocations goes to infinity. We study the asymptotic behaviour of our nonlinear dislocation energy as the length of the Burgers vector tends to zero via Γ-convergence. In the limit we obtain strain-gradient models for plasticity of the same form as the limit energies derived from a quadratic dislocation energy. Our result, however, is obtained by starting from a more physical model.

References

[1] L. Scardia and C.I. Zeppieri: *Line-tension model for plasticity as the* Γ*-limit of a nonlinear dislocation energy*, SIAM J. Math. Anal., to appear.

Applications of equivalence and symmetry point transformations in studying PDEs

Olena Vaneeva

After brief review of basic notions and properties of Lie symmetries, the statement of group classification problem will be given [1,2]. The role of equivalence and admissible transformations in successful solving such problems will be shown using the illustrative examples of second order (reaction-diffusion) and/or third order (mKdV type) evolution equations [3,4].

References

[1] P.J. Olver: *Applications of Lie Groups to Differential Equations*, Springer-Verlag, 1986.

[2] L.V. Ovsiannikov: *Group analysis of differential equations*, Academic Press, 1982.

[3] R.O. Popovych and O.O. Vaneeva: *More common errors in finding exact solutions of nonlinear differential equations. I*, Commun. Nonlinear Sci. Numer. Simul. 15 (2010), 3887-3899.

[4] O.O. Vaneeva, R.O. Popovych, and C. Sophocleous: *Enhanced group analysis and exact solutions of variable coefficient semilinear diffusion equations with a power source*, Acta Appl. Math. 106 (2009), 1-46.