Young Women in PDEs

Bonn, 10–12 May, 2012

Poster Session: Abstracts

Institute for Applied Mathematics
University of Bonn

Bonn International Graduate
School in Mathematics

Bounded solutions for a quasilinear singular problem with non linear Robin boundary conditions

Imen Chourabi and Patrizia Donato

In this work, we study the existence of bounded positives solutions for a quasilinear singular problem with nonlinear Robin conditions. The nonlinear term is singular with respect to the solution and has a quadratic growth with respect to its gradient.

The corresponding singular case with Dirichlet conditions has been proved by F. Murat and D. Giachetti in [3] and the quasilinear case (without the nonlinear term) with nonlinear Robin conditions has been studied by B. Cabarrubias and P. Donato in [2]. The existence of a solution for nonsingular quadratic growth in the gradient and Dirichlet conditions has been proved in [1].

As in [3], we prove the result by approximation. Here, the approximate problem does not present the singularity and is bounded with respect to the gradient. In a first step we prove the existence of a bounded solution of this problem. As far as we know, this is new in the literature, due to the presence of the nonlinear boundary condition. To do that, we apply the Schauder fixed point Theorem and use some results from [2].

Then, using some equi-integrability arguments we pass to the limit in the approximate problem and obtain at the limit a solution of our problem.

The results will be the object of an article (to appear).

References

[1] L. Boccardo, F. Murat and J.L. Puel, *Existence de solutions faibles pour des équations elliptiques quasi-linéaires à croissance quadratique*, Nonlinear Partial Differential Equations and their Applications, Collège de France Seminar, H. Brezis & J.L. Lions eds, Research Notes in Math. 84, , 19-73 (Pitman, London 1983).

[2] B. Cabarrubias and P. Donato, *Existence and Uniqueness for a Quasilinear Elliptic ProblemWith Nonlinear Robin Conditions,* Carpathian J. Math., 27 (2) (2011), 173-184.

[3] D. Giachetti, F. Murat, *Elliptic problems with a lower order terms having singular behaviour*, Boll. Unione Mat. Ital. (9) 2 (2009), no. 2, 349-370.

Mathematical modeling and simulations of parabolic trough power plants

Federica Di Michele and Ingenuin Gasser

Parabolic trough power plants is a technology for utilizing solar energy in large power plants. These use curved, long, parabolic mirrors that focus the sun's direct beam radiation on a linear receiver tube located on the focus of the parabola. This tube contains a heat transfer fluid (HTF), that is heated and it is used to generated superheated steam. At the final step the superheated steam is transformed into electricity by using a reheat steam turbine-generator. Different kinds of HTFs are now available:

(1) Oil
(2) Ionic fluid
(3) Water and steam

In this talk we show a simple mathematical model for heat and mass transfer in parabolic trough power plants. We also present a suitable numerical code, by use of which we compare the performance of the different kinds of transfer fluid, in order to understand which provides the best performance.

Modelling, Asymptotic Analysis and Simulation of Gas Dynamics in Chimneys and Cooling towers

Elisabetta Felaco and Ingenuin Gasser

We start with a basic example in which we describe the motion of flue gases produced by combustion trough a chimney.

We use a one dimensional fully compressible model, considering an adapted version of the Euler equations of gas dynamics and after a standard scaling, we encounter two small parameters, the Mach number M and the Froud number Fr, so that we have at least two possible choices for asymptotic analysis: the Boussinesq approximation and the small Mach number limit.

We find that the first one is not applicable in our case while the second lead us to a simpler system of equations on which we perform numerical simulations.

The scheme that we use is simple, and it is faster then the corresponding simulations on the full set of compressible equations.

Once analysed in details this basic model, we apply the same techniques to a more sophisticated problem, such as that of cooling towers. We focus on natural draft hybrid cooling towers, were the cooling effect is due both to the flow of fresh air and to the evaporation of cooling water.

In this case we have to deal with a complex set of equations describing the dynamics of water and of the mixture of dry air and water vapour, so that the asymptotic simplification becomes essential to solve the problem.

Asymptotic formula of Stokes problem due to a small perturbation of the domain

Luong Thi Hong Cam and Daveau Christian.

Let $\Omega \subset \mathbb{R}^2$ be a bounded open domain with $\partial\Omega$ is of class C^2. We consider the Stokes system with inhomogeneous boundary condition $P(g, \Omega)$:

$$\begin{cases} -\Delta u + \nabla p = \lambda u & \text{in} \quad \Omega \\ \nabla \cdot u = 0 & \text{in} \quad \Omega \\ u = g & \text{on} \quad \partial\Omega. \end{cases} \tag{1}$$

We consider the problem (1) with a perturbed geometry $\partial\Omega_\delta$ of $\partial\Omega$ such that

$$\partial\Omega_\delta = \{\widetilde{x} : \widetilde{x} = x + \delta\rho(x)\nu(x), x \in \partial\Omega\},$$

where $\nu(x)$ is the outward normal vector on $\partial\Omega$, $\rho(x)$ is of class C^1 with $\|\rho\|_{C^1} \leq 1$, and g_δ is the continuous extension of g that satisfies $g_\delta = g$ on $\partial\Omega$, and with $\delta \ll 1$. We consider the perturbed problem $P(g_\delta, \Omega_\delta)$:

$$\begin{cases} -\Delta u_\delta + \nabla p_\delta = \lambda u_\delta & \text{in} \quad \Omega_\delta \\ \nabla \cdot u_\delta = 0 & \text{in} \quad \Omega_\delta \\ u_\delta = g_\delta & \text{on} \quad \partial\Omega_\delta. \end{cases} \tag{2}$$

We verify the stability of the solution u_δ with respect to δ : There is a positive constant C such that

$$\|u_\delta - u\|_\infty \leq C\delta\|g\|_{L^2(\partial\Omega)},$$

and then derive an asymptotic expansion of $u_\delta - u$ in terms of δ by using layer potential techniques.

References

[1] H. Ammari, H. Kang, M. Lim, and H. Zribi. *Conductivity interface problems. Part I: Small perturbation of an interface*, to appear in Trans. Amer. Math. Soc.

[2] Wolfgang Borchers, Werner Varnhorn. *On the boundedness of the Stokes semigroup in two-dimension exterior domains.* Math. Zeitschrift, Volume 213, Number 1, 275-299.

[3] Habil Ammari. *An introduction to Mathematics of Emerging Biomedical Imaging.* Springer, 2008.

Removable singularities of solutions of elliptic equations with nonstandard grow conditions

Yuliya V. Namlyeyeva and Igor I. Skrypnik

We study solutions of general quasilinear elliptic equation

$$- \operatorname{div} A(x, u, \nabla u) = B(x, u, \nabla u), \quad x \in \Omega \setminus \{x_0\} \tag{3}$$

where $x_0 \in \Omega \subset \mathbb{R}^n$, $n \geq 2$. Here $A(x, u, \xi)$, $B(x, u, \xi)$ satisfy the Caratheodory conditions and the following inequalities are valid:

$$A(x, u, \xi)\, \xi \geq a_0 f(|\xi|)|\xi|^2 - a_1 f(|u|)|u|^2 - a_1,$$

$$|A(x, u, \xi)| + |B(x, u, \xi)| \leq a_2 \left(f(|\xi|)|\xi| + f(|u|)|u| + 1 \right)$$

for every $x \in \Omega$, $u \in \mathbb{R}^1$, $\xi \in \mathbb{R}^n$, where a_1, a_2 are some positive constants. The function $f(t)$ defined for $t \geq 0$ is a continuous nondecreasing function satisfying the following inequalities:

$$\nu_1\, t^{p-2} - \nu_1 \leq f(t) \leq \nu_2\, t^{q-2} + \nu_2,$$

$$f(2\,t) \leq \nu_3 f(t),$$

with some positive constants ν_1, ν_2, ν_3 and p, q such that

$$1 < p \leq q < \frac{n-1}{n-p}\, p, \quad p < n.$$

Moreover, for the function $g(t) = t\, f(t)$ satisfying $g(0) = 0$, the following inequality is valid

$$\int_0^t g(s)\, ds \geq \nu_4\, g(t)\, t,$$

for any $t > 0$, and some constant $0 < \nu_4 \leq 1$.

We establish the precise pointwise condition for removable isolated singularities of solutions to equation (3). This study is a continuation of the previous research of the authors [1,2].

References

[1] Namlyeyeva, Yu.V., Shishkov, A.E., Skrypnik, I.I.: *Isolated singularities of solutions of quasilinear anisotropic elliptic equations*, Advanced Nonlinear Studies, **6**:617–641, 2006.

[2] Namlyeyeva, Yu.V., Shishkov, A.E., Skrypnik, I.I.: *Removable isolated singularities for solutions of doubly nonlinear anisotropic parabolic equations*, Applicable Analysis, **89**(10):1559–1574, 2010.

Symmetric Discontinuous Galerkin Formulation For Maxwell's equations

Azba Riaz and Christian Daveau.

In this paper, a new symmetric discontinuous Galerkin formulation for the time-dependent Maxwell's equations with superconductive boundary has been proposed. Its hp analysis is carried out and error estimates that are optimal in the meshsize h is obtained. Some numerical results are given to confirm the convergence rates as a function of the meshsize. In [3], the discontinuous Galerkin method with solutions that are exactly divergence-free inside each element, is developped for numerically solving the Maxwell equations. Here, we consider a symmetric interior penalty discontinuous Galerkin method to approximate in space an initial boundary value problem derived from Maxwell's equations in stable medium with supraconductive boundary.

$$\frac{\partial^2 u}{\partial t^2} + c^2 \nabla \times (\nabla \times u) = f, \ \nabla . u = 0 \tag{1}$$

$$n \times u(x,t) = 0 \text{ on } \partial\Omega \times I, \ u(x,0) = u_0, \ \frac{\partial u}{\partial t}(x,0) = u_1 \text{ on } \Omega \tag{2}$$

Here Ω is a convex polyhedron,

$$I = [0; t^*] \subset \mathbb{R}, u_0 \text{ and } u_1 \text{ are in } H_0(\nabla \times, \Omega) \cap H(\nabla ., \Omega) \text{ and } f \text{ is defined on } \Omega \times I.$$

In this paper we used the same notations for spaces as in [1]. In order to derive a weak formulation of (1)-(2), we note that formulas (1)-(2) in [1] implies for any u with $\nabla \times u \in H(\nabla \times, \Omega)$

$$c^2 < \nabla \times (\nabla \times u), v >= c^2 < \nabla \times u, \nabla \times v > + a(u,v)$$

where we have denoted

$$a(u,v) = c^2 < n \times (\nabla \times u), v > - c^2 \sum_{e \subset F_h^I} < [v]_T, \{\nabla \times u\} >_e$$

Now, we introduce the penalty term via the form

$$J^\sigma(u,v) = \sum_{e \subset F_h^I} < \sigma [u]_N, [v]_N >_e + \sum_{e \subset F_h} < \sigma [u]_T, [v]_T >_e \quad u, v \in H^1(\nabla \times, \Pi)^3$$

where $\sigma = kp^2/h$ is a stabilization parameter and k is a constant supposed ≥ 1. We also defined

$$A(u,v) = c^2(\nabla \times u, \nabla \times v) + a(u,v) - a(v,u) + J(u,v) \text{ where } J(u,v) = (\nabla . u, \nabla . v)$$

So our formulation is $B(u,v) = A(u,v) + J^\sigma(u,v)$.

References

[1] A. Zaghdani and C. Daveau,*Two new discrete inequalities of Poincare-Friedrichs on discontin- uous spaces for Maxwell's equations.*,C. R. Acad. Sci. Paris. Ser.I, 342, pp 29-32, 2006.

[2] P. Houston, C. Schwab and E. Souli ,*Discontinuous hp-finite element methods for advection- diffusion-reaction problems.*,SIAM J. Numer. Anal., 39, pp 2133-2163, 2002.

[3] B. Cockburn, F. Li, C.-W Shu,*Locally divergence-free discontinuous Galerkin method for Maxwell equations.,Journal of Computational Physics.*,Vol. 194 , pp 588-610, 2004.

On solutions of some boundary problems for inhomogeneous PDE with nondense defined operator in special spaces of entire functions

Tetiana Stulova, and Sergey Gefter

In the talk we suppose to tell about the two abstract theorems, relating to theory of differential equations in a Banach space and about its applications to solution of some boundary problems for inhomogeneous PDE with nondense defined operator.

Here we illustrate one of the theorem and one of the example.

Let E be a Banach space and $A : D(A) \to E$ be closed invertible linear operator on E (where its domain of definition $D(A)$ can be not dense).

Theorem. *Let $f(z)$ be an E-valued entire function of zero exponential type. Then the differential equation*

$$w' = Aw + f(z) \qquad (1)$$

has the unique entire solution of zero exponential type $w(z) = -\sum\limits_{n=0}^{\infty} A^{-(n+1)} f^{(n)}(z)$, at that this solution continuously dependent on f in the nature topology of space of all entire E-valued functions of zero exponential type.

Example. Let $E = C[0,\ 1]$, $\quad A = \frac{d^2}{dx^2}$, $\quad D(A) = \{u \in C^2[0,\ 1] : u(0) = u(1) = 0\}$. Then operator A is invertible, $(A^{-1}h)(x) = \int\limits_0^1 G(x,\ y)\ h(y)dy$, where G is the Green function of corresponding boundary problem. In this case

$$\left(A^{-(n+1)}h\right)(x) = \int\limits_0^1 G_{n+1}(x,\ y)\ h(y)\,dy,\ \text{where } G_1(x,\ y) = G(x,y)\,,$$

$$G_{n+1}(x,\ y) = \int\limits_0^1 G_n(x,\ s)\ G(s,\ y)\,ds.$$

In this example by transition to real axes Equation (1) has the form of the heat equation on $(0,\ 1)$ with zero boundary conditions

$$\begin{cases} \frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + f(t,x)\,, & t \in R,\ x \in (0,\ 1) \\ w(t,\ 0) = w(t,\ 1) = 0 \end{cases}$$

If $f(t,x) = \sum\limits_{n=0}^{\infty} c_n(x)\ t^n$, where $c_n \in C[0,1]$ and $\lim\limits_{n\to\infty} \sqrt[n]{n!\,\|c_n\|} = 0$, then Equation (1) has the solution

$$w(t,x) = -\sum_{n=0}^{\infty} \int\limits_0^1 G_{n+1}(x,y)\,\frac{\partial^n f}{\partial\, t^n}(t,y)\,dy.$$

One Problem of Optimal Forest Harvesting

Victoria Veshchinskaya and Elena Rovenskaya

In this work we consider the dynamic system described by the transport equation with constant coefficients

$$x_t(t,l) + gx_l(t,l) = -\mu x(t,l) \quad (t \in [0,T], l \in [0,L]). \tag{4}$$

Here $x(t,l) \in R^1$ $(t \in [0,T], l \in [0,L])$, $T > 0$, $L > 0$, $g > 0$ and $\mu > 0$ are given. System (4) is assumed to satisfy the general initial condition and the nonlocal boundary condition:

$$x(0,l) = x_0(l) \quad (l \in [0,L]); \quad gx(t,0) = p(t) + \beta\int_0^L x(t,l)dl \quad (t \in [0,T]), \tag{5}$$

where $x_0(\cdot) : [0,L] \mapsto R_1^+$, $p(\cdot) : [0,T] \mapsto R_1^+$ and $\beta > 0$ are given. The dynamic system (4) – (5) arises from the forest life cycle modeling [1,2].

Theorem 1 Let $x_0(\cdot)$ and $p(\cdot)$ be sectionally continuously differentiable functions and $gx_0(0) = p(0) + \beta\int_0^L x_0(l)dl$. Then the unique solution of the system (4) – (5) exists and takes form

$$x(t,l) = \begin{cases} a\left(t - \frac{l}{g}\right)e^{-\frac{\mu}{g}l}, & l \in [0,gt] \\ x_0(l-gt)e^{-\mu t}, & l \in [gt,L] \end{cases} \quad (t \in [0,T]), \tag{6}$$

where $a(t) = \frac{1}{g}p(t) + \frac{\beta}{g}e^{-\mu t}\int_0^{L-gt} x_0(s)ds + \frac{\beta}{g}e^{(\beta-\mu)t}\int_0^t e^{(\mu-\beta)s}p(s)ds +$
$\frac{\beta^2}{g}e^{(\beta-\mu)t}\int_0^t e^{-\beta s}\int_0^{L-gs} x_0(l)dlds$ $(t \in [0,T])$.

Let $x_0(\cdot) : x_0(l) = \bar{x}(l)$ for $l \in [0, \bar{l})$, and $x_0(l) = (1-\alpha)\bar{x}(l)$ for $l \in [\bar{l}, L]$, $\bar{x}(l) = \gamma e^{-\frac{\mu}{g}l}$, $\gamma = \bar{x}(0) > 0$. Let $p(\cdot) \in C$, where C denotes the set of sectionally continuously differentiable functions possesses the values in $[0, P]$. Let $\alpha \in [\alpha^-, \alpha^+]$ and $\bar{l} \in [\bar{l}^-, \bar{l}^+]$ $(0 \leq \alpha^- < \alpha^+ \leq 1,\ 0 \leq \bar{l}^- < \bar{l}^+ \leq L)$. Let us define the functionals on $C \times [\alpha^-, \alpha^+] \times [\bar{l}^-, \bar{l}^+]$: $B(p(\cdot), \alpha, \bar{l}) = \alpha\int_{\bar{l}}^L \bar{x}(l)dl - h\int_0^T p(t)dt$; $E(p(\cdot), \alpha, \bar{l}) = \int_0^L x(T, l)dl$ and $U(p(\cdot), \alpha, \bar{l}) = B(p(\cdot), \alpha, \bar{l}) + \sigma E(p(\cdot), \alpha, \bar{l})$, where $\sigma \geq 0$ is a fixed weighting factor. In this work we maximize the functional $U(p(\cdot), \alpha, \bar{l})$ in the extreme case $L \to \infty$.

References

[1] Hritonenko N., Yatsenko Yu., Goetz R. and Xabadia A.: *Maximum principle for a size-structured model of forest and carbon sequestration management*, Applied Mathematics Letters 21: 1090-1094, 2008.

[2] Hritonenko N., Yatsenko Yu., Goetz R. and Xabadia A.: *A bang-bang regime in optimal harvesting of size-structured populations*, Nonlinear Analysis 71: 2331-2336, 2009.

On existence for a system modeling water-gas flow in porous media in a fully equivalent global pressure formulation

Anja Vrbaški, Brahim Amaziane, and Mladen Jurak

Immiscible compressible two-phase flow through porous media under isothermal condition is described by a new model, developed in [1], [2], which employs the global pressure and the non-wetting phase saturation as primary unknowns. The resulting equations are written in a fractional flow formulation which leads to a degenerate coupled system consisting of a nonlinear parabolic equation (the global pressure equation) and a nonlinear convection-diffusion one (the saturation equation). This new formulation is fully equivalent to the original phase equations formulation without any simplifying assumptions.

In this work, we consider a system in which each component only appears in one of the phases with no mass transfer between the phases. Furthermore, we assume that the wetting phase is incompressible and the non-wetting phase is compressible, such as water and gas in the context of gas migration through engineered and geological barriers for a deep repository of nuclear waste. The non-homogeneous Dirichlet and Neumann conditions are imposed on the boundary of the domain. Under some physically relevant assumptions on the data, as in [3], as well on the boundary data, we prove the existence of weak solutions of the coupled and degenerate system by using an appropriate regularization and a time discretization. A priori estimates are obtained using suitable test functions, and a compactness result needed to pass to the limit in the nonlinear terms is established.

References

[1] B. Amaziane, M. Jurak: *A new formulation of immiscible compressible two-phase flow in porous media*, C. R. Mecanique, 336:600-605, 2008.

[2] B. Amaziane, M. Jurak, A. Žgaljić Keko: *Modeling and Numerical Simulations of Immiscible Compressible Two-Phase Flow in Porous Media by the Concept of Global Pressure*, Transport in Porous Media, 84:133-152, 2010.

[3] B. Amaziane, M. Jurak, A. Žgaljić Keko: *An Existence Result for a Coupled System Modeling a Fully Equivalent Global Pressure Formulation for Immiscible Compressible Two-Phase Flow in Porous Media*, J. Differential Equations, 250:1685-1718, 2011.