V5B6 Advanced Topics in Analysis and Calculus of Variations: Quantitative Isoperimetric-type Inequalities

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Abstract

The classical isoperimetric inequality is one of the most famous and prominent examples of a geometric inequality, characterizing Euclidean balls as the unique minimizers of the perimeter functional among sets of fixed volume. A subsequent natural question is related to its *stability*, in the sense of quantifying how much an arbitrary set deviates from being optimal in terms of a suitably defined *isoperimetric deficit*. In recent years, significant progress has been made towards sharp quantitative versions of the isoperimetric, as well as other related geometric and functional inequalities, with applications ranging from Calculus of Variations, Differential Geometry, Geometric Flows, but also to Mathematical Physics.

The object of the course is to explore the principles behind establishing the sharp quantitative isoperimetric inequality, studying in detail the three main, conceptually distinct approaches that lead to its proof. We present:

(i) Variational methods for stability involving techniques from nonlinear PDE and minimal surface theory.

(ii) Quantitative symmetrization techniques based on the original proof in the class of sets of finite perimeter.

(iii) An alternative approach based on optimal transport theory.

Throughout the course, we emphasize the ideas, technical tools, and geometric intuition behind each method, highlighting their strengths, limitations, and possible applicability to other related variational problems.

Prerequisites

Analysis I-III, Measure theory and basics of PDE (*e.g.* PDE-Functional Analysis) will be helpful.

References

[1] M. Cicalese, G.P. Leonardi,. *A selection principle for the sharp quantitative isoperimetric inequality* Arch. Ration. Mech. Anal. **206** (2012), 617-643.

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[3] N. Fusco. *The quantitative isoperimetric inequality and related topics.* Bulletin of Mathematical Sciences, **5** (2015), 517–607.

[4] N. Fusco, F. Maggi, A. Pratelli. *The sharp quantitative isoperimetric inequality* Ann. of Math. **168** (2008), 941–980.

[5] F. Maggi. Sets of finite perimeter and geometric variational problems. An introduction to geometric measure theory. *Cambridge Studies in Advanced Mathematics* **135**, Cambridge University Press (2012).