

Quantitative Isoperimetric-Type Inequalities

Summer Semester 2026

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Report of the lectures.

Lecture I.1 (on 13.04.2026): the Dido's problem, Hurwitz's proof, De Giorgi's formulation of the isoperimetric problem, Bennesen-type inequalities, the isoperimetric deficit, Fuglede's result and ideas, Fraenkel's asymmetry, Hall's conjecture.

Lecture I.2 (on 14.04.2026): outer measures, Lebesgue measure, measurable sets, Borel and Borel regular measures, Radon measures, inner and outer approximations, restrictions and support, Hausdorff measure, properties and example, $\mathcal{H}^1 = \text{length}$.

Lecture I.3 (on 20.04.2026): Riesz's Theorem, vector-valued outer measures, weak* convergence and compactness, Besicovitch covering, Lebesgue-Besicovitch differentiation, density of sets, the Area formula for linear maps.

Lecture I.4 (on 21.04.2026): set of Jacobian zero, Lipschitz linearization lemma, proof of the Area formula, $\mathcal{H}^k$ -rectifiable sets, decomposition into regular Lipschitz images, existence of an approximate tangent space for regular Lipschitz images.

Lecture I.5 (on 27.04.2026): approximate tangent space for $\mathcal{H}^k$ -rectifiable sets, sets of finite perimeter, Gauss-Green measure, perimeter and relative perimeter, the role of the divergence theorem, examples, relations with BV functions.

Lecture I.6 (on 28.04.2026): local convergence of sets, lower semicontinuity of the perimeter, examples, topological boundary and support of Gauss-Green measure, regularization via mollification, coercivity of the perimeter.

Lecture I.7 (on 04.05.2026): the coarea formula, examples and discussion, approximation of sets of finite perimeter with smooth (and bounded) sets, the constraint isoperimetric problem, reduction to minimization among smooth sets.

Lecture I.8 (on 05.05.2026): optimality of the ball (not proved yet), proof of the isoperimetric inequality, comments and discussion (the planar case), the reduced boundary and (measure-theoretic) outer normal vector, blow-ups of sets of finite perimeter and their tangential properties, De Giorgi's structure theorem.

Lecture IV.1 (on 29.06.2026): definition of 1-dimensional slices and Steiner symmetral, volume invariance, slicing of perimeter, definition of sets with polyhedral boundary, Steiner inequality, proof and examples.

Lecture IV.2 (on 30.06.2026): complete proof of the isoperimetric problem (via Steiner symmetrization), the proof of quantitative isoperimetric inequality via symmetrization techniques, road-map of the proof, definition of n -symmetric and axis-symmetric sets, property of Fraenkel asymmetry for n -symmetric sets, deviation of two shifted balls (without proof).

Lecture IV.3 (on 13.07.2026): energetically convenient symmetrization across one coordinate plane, reduction to n -symmetric sets, Schwarz symmetrization and related properties, coarea formula for rectifiable sets, Schwarz inequality.

Lecture IV.4 (on 14.07.2026): avoiding concentration at the poles and in spikes (only ideas), reduction to axis-symmetric sets, asymmetry controlled by deviation of central slices, estimate on axis-symmetric sets (only ideas), proof of the quantitative isoperimetric inequality in full generality.