UNIVERSITÄT BONN

Universität Bonn
Institut für Angewandte Mathematik

Γ-convergence of integral functionals

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Content: in mathematics and related fields, we are often faced with problems depending on a parameter that may come from an approximation or a discretization procedure or may be constitutive of the problem itself, e.g. a geometric quantity. Sometime the analysis of these problems is challenging and it can be useful to rather study their limit as the dependence on the parameter disappears.

The Γ-convergence is a very general notion of convergence of functionals. Since its introduction by De Giorgi, has been recognized as the natural convergence of variational problems. Indeed, provided the right set of assumptions, global minima of Γ-converging functionals converge in turn to the global minimum of the Γ-limit and the same holds for global minimizers.

In the first part of the course, we focus on the abstract notion of Γ-convergence of a family of functionals. We will move our attention to integral functionals defined in Sobolev spaces and deal with the theory of Relaxation. In the second part, we will attack pivotal problems with this new tool: homogenization, discrte-to-continuum limit, phase-transition problems, (approximation of) free-discontinuity problems. A preliminary list of contents:

- lower-semicontinuity and Γ-convergence;
- notions of convexity;
- Relaxation of integral functionals;
- periodic Homogenization;
- gradient theory of phase-transitions.

Prerequisites: knowledge of Sobolev spaces, some Functional Analysis and the bases of Calculus in $\mathbb{R}^n$ are needed.

Format: "Selected Topics", i.e. 2 hours per week.

Literature: we will mainly follow the books

- A. Braides and A. Defranceschi, *Homogenization of Multiple Integrals*, Oxford Univ. Press, Oxford (1998).
- A. Braides, *Γ-convergence for Beginners*, Oxford Univ. Press, Oxford (2002).

Another interesting reading is Dal Maso's book *An Introduction to Γ-convergence*, Birkhäuser, Boston (1993).