

Universität Bonn Institut für Angewandte Mathematik

Γ-convergence of integral functionals Summer Semester 2024 Dr. Antonio Tribuzio

Report of the lectures.

Lecture 1 (on 15.04.2024): quick overview of the course, CalcVar and Direct Method, the meaning (and need) of Γ -convergence, example of a homogenization problem.

Lecture 2 (on 22.04.2024): upper and lower limits, lower semicontinuity (definitions and properties), definition of Γ -convergence, example on the real line, upper and lower Γ -limits.

Lecture 3 (on 29.04.2024): lower semicontinuity of Γ -limits, stability under continuous perturbations, Γ -limit of constant and monotone sequences, coerciveness notions, the fundamental Theorem of Γ -convergence.

Lecture 4 (on 06.05.2024): Direct Method with relaxation, compactness of Γ -convergence, integral functionals in Lebesgue spaces, necessary conditions for (seq.) weak^{*} lower-semicontinuity.

Lecture 5 (on 10.05.2024): sufficient conditions for (seq.) weak lower-semicontinuity, strong lower-semicontinuity, (seq.) weak(*) lower-semicontinuous envelope.

Lecture 6 (on 13.05.2024): weak coerciveness, equivalence between (seq.) weak lower-semicontinuity and weak lower-semicontinuity under growth conditions, Γ -convergence of integral functionals in Lebesgue spaces.

Lecture 7 (on 27.05.2024): integral functionals (on the gradient) in Sobolev spaces, weak convergence of oscillating piecewise affine functions, necessary conditions for (seq.) weak lower-semicontinuity, notions of polyconvexity, quasiconvexity and rank-1 convexity.

Lecture 8 (on 03.06.2024): quasiconvexity (and growth conditions) implies rank1-convexity, characterization of (seq.) weak lower-semicontinuity in Sobolev spaces, equivalence between (seq.) weak lower-semicontinuity and L^p -strong lower-semicontinuity under growth conditions.

Lecture 9 (on 10.06.2024): localization method of Γ -convergence, integral representation of functionals in Sobolev spaces, the case of homogeneous functionals, fundamental estimate.

Lecture 10 (on 17.06.2024): inner regularity and subadditivity of (localized) Γ -limit and Γ -limsup, compactness of (localized) Γ -limit and its integral representation.

Lecture 11 (on 24.06.2024): stability of Γ -convergence under compatible boundary conditions and equi-coerciveness, definition of quasiconvexification (and some properties), Γ -convergence and relaxation of homogeneous functionals, example of 1-dimensional problem.

Lecture 12 (on 01.07.2024): setting of periodic homogenization, cluster points (wrt Γ -convergence) are homogeneous integral functionals, asymptotic homogenization formula, homogenization formula on periodic functions.

Lecture 13 (on 08.07.2024): cell formula for convex homogenization, homogenization of nonlinear PDEs, a one-dimensional example, homogenization of quadratic forms.

Lecture 14 (on 15.07.2024): gradient theory of phase transition, physical motivation, Γ -convergence of Modica-Mortola energy.