Sample Exam Questions

V5F2 Selected Topics in Probability Theory – Discrete Random Matrices

Winter Semester 2024/25, Instructor: Lisa Sauermann

This list is meant to give a flavor of type of questions in exam. Not all questions in the exam will be from this list, but some questions from the list may be asked in the exam.

- What does the Erdős-Littlewood-Offord Theorem state?

- What is the proof idea for the Erdős-Littlewood-Offord Theorem?

- How can the Erdős-Littlewood-Offord be used to bound the singularity probability of a random $n \times n$ matrix with independent uniform $\{1, -1\}$-entries?

- What bounds does this proof give roughly?

- What bound did Tikhomirov prove for the singularity probability of a random $n \times n$ matrix with independent uniform $\{1, -1\}$-entries? Is his bound tight?

- Tikhomirov's proof splits into two parts. What are these parts? In particular, what are compressible vectors and incompressible vectors?

- What is the strategy for the compressible part?

- What is the strategy for the incompressible part?

- How is the threshold $\mathcal{T}(x, L)$ (for a unit vector x and a parameter $L > 0$) defined?

- What can you say about the matrix norm of a random $n \times n$ matrix with independent uniform $\{1, -1\}$-entries? How big is it typically? Does the same hold for a random $n \times n$ matrix with independent uniform $\{0, 1\}$-entries?

- What is the Lévy concentration function of a (real) random variable?

- What is a net argument? Can you give an example where such an argument is used?