S4F3 Graduate Seminar on Applied Probability
Probabilistic Combinatorics

Winter Semester 2026/27

Time and location: **Wednesdays 12-14** in seminar room N0.007

Preliminary meeting: Wednesday October 14 (2026) at 12:15 in seminar room N0.007
(in the first week of classes at the usual time of the seminar)

Instructors:
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Prerequisites:
basic knowledge of probability theory, combinatorics and graph theory (on the level of introductory courses)

The topic of this graduate seminar is "Probabilistic Combinatorics", the study of combinatorial problems using methods from probability theory. The talks in this seminar will cover different results in this area, and many of the talks are on recent research papers. As a prerequisite, students should be familiar with basic probability theory, combinatorics and graph theory, on the level of introductory courses.

In order to participate in this seminar, please come to the preliminary meeting on October 14, where the talks will be assigned. If you cannot come to the preliminary meeting, but would like to participate in the seminar, you can send an email to one of the instructors (maybe a solution can be found).

Every student is required to meet with the instructors at least a week prior to their talk to discuss the seminar talk (email Eden to schedule). Students must bring detailed notes for the talk to the meeting.

It is very important that the mathematical content of the talks is not only correct, but also comprehensible for the audience. To this end, it is helpful to give examples for the relevant definitions and notions, and to explain the overarching proof ideas clearly (to make it easier to follow the details of the proof). The grading is based on mathematical correctness as well as on the presentation style. Interactions with the audience (e.g. by asking questions to the audience) are highly encouraged.

The talk should be planned for around 70 minutes (to allow for time for questions during and after the talk).

The seminar will not meet on December 2 (Dies Academicus) and also not on December 23, December 30 and January 6 (Winter Break).

The following is a list of possible talk topics (depending on the number of students, not all topics will be covered, and the order of the talks can also be changed to some extent):

- The crossing lemma for graph drawings and applications [7].
- Lower bounds for Ramsey numbers. Start with the classical result for the Ramsey number [2, Section 1.1] and proceed with the recent lower bound for the multicolour Ramsey number $r(3;k)$ [13, Chapter 9].
- Discrepancy: A proof of the Komlós conjecture [6] (you might prefer the exposition in [8] for a shorter treatment).
- Algorithmic Lovász Local Lemma [11].

- Expectation thresholds: a proof of the Kahn–Kalai conjecture [14].
- The thresholds of clique factors [15].
- Random algebraic constructions in extremal graph theory [4].
- Dependent random choice and the Balog–Szemeredi–Gowers theorem [5, Chapters 2 and 5].
- Entropy methods [16, Chapters 1 and 5].
- The average degree of graphs without a K_t-minor [1, 3].
- A proof of the random Ramsey theorem using the container method [12] (you may also consult [9]).
- Talagrand's inequality and the longest increasing subsequence in a random permutation [2, Chapters 7.6 and 7.7].
- Euclidean travelling salesman [17, Section 9.6].
- Bourgain's embedding theorem [10, Chapters 1 (definitions) and 4 (proof)].

References

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[3] B. Bollobás, P. Catlin, and P. Erdős, Hadwiger's conjecture is true for almost every graph. *European Journal of Combinatorics*. **1**, 195-199, 1980. Available at `https://www.renyi.hu/~p_erdos/1980-10.pdf`

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[6] S. Guo, E. Fang, and J. Lu, Vector balancing in polynomial time, 2026. Preprint version available at `https://arxiv.org/pdf/2609.23540`

[7] M. Hoffmann, *Chapter 3: Crossings*, Lecture Notes for Geometry: Combinatorics & Algorithms, ETH Zürich, 2025. Available at `https://ti.inf.ethz.ch/ew/courses/Geo25/lecture/gca25-3.pdf`.

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