

S4F5 Graduate Seminar on Interacting Random Systems

Probabilistic Combinatorics

Winter Semester 2025/26

Time and location: **Fridays 12-14** in seminar room N0.003

Preliminary meeting: July 21 (2025) at 10:30 in seminar room N0.003

Prerequisites:

basic knowledge of probability theory, on the level of an introductory course

Instructors:

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Marcus Kühn

The topic of this graduate seminar is “Probabilistic Combinatorics”, the study of combinatorial problems using methods from probability theory. The talks in this seminar will cover different results in this area, and many of the talks are on recent research papers. As a prerequisite, students should be familiar with basic probability theory, on the level of an introductory course. Relevant concepts from combinatorics will be introduced during the seminar (prior knowledge of basic combinatorial concepts such as sets and graphs is helpful, but not required). No prior exposure to probabilistic combinatorics is expected. The seminar may also be suitable for advanced undergraduate students interested in probability theory or combinatorics (or both).

In order to participate in this seminar, please come to the preliminary meeting on July 21, where the talks will be assigned. If you cannot come to the preliminary meeting, but would like to participate in the seminar, you can send an email to one of the instructors (maybe a solution can be found).

Every student is required to meet with the instructors at least one week in advance of the talk to discuss the student’s talk outline (the student is required to bring detailed notes for the talk to the meeting).

It is very important that the mathematical content of the talks is not only correct, but also comprehensible for the audience. To this end, it is helpful to give examples for the relevant definitions and notions, and to explain the overarching proof ideas clearly (to make it easier to follow the details of the proof). The grading is based on mathematical correctness as well as on the presentation style. Interactions with the audience (e.g. by asking questions to the audience) are highly encouraged.

The talk should be planned for around 70 minutes (to allow for time for questions during and after the talk).

The following is a list of possible talk topics (depending on the number of students, not all topics will be covered, and the order of the talks can also be changed to some extent):

- Basic graph theoretic notions, proof of existence of high-girth graphs with large chromatic number, asymptotic notation, Chernoff bound (with proof)
[12, Section 1.2 until top of page 10], [1, “Probabilistic Lens” after Chapter 3 on p. 43–44], [12, Section 2.1 until Theorem 2.8]
- Ramsey Numbers: Sketch of inductive proof of simple upper bound, probabilistic proof of lower bounds for Ramsey numbers for two colors, and for more colors
[1, Section 1.1], [15, Section 1], [notes from previous course of instructor]
- New upper bounds on (multi-color) Ramsey numbers [3]
- Entropy and applications
[1, Section 15.7], [9, Sections 2.2, 2.3, and 3.1]

- Constant lower bound for the Union-Closed Conjecture [10] or [16]
- Random graphs: containment of small subgraphs and clique number [1, Sections 4.3 to 4.5]
- Dependent random choice [7, Sections 2 and 3]
- Martingales and applications, in particular to the chromatic number of a random graph [1, Sections 7.1 to 7.4]
- Non-concentration of the chromatic number of a random graph [11]
- Erdős Covering Systems [4]
- The Rödl nibble [1, Section 4.7] or [8]
- Introduction to the differential equation method [6, Section 3]
- Random greedy hypergraph matching [5]
- A general differential equation method theorem [18]
- Proof of the Kahn–Kalai Conjecture (and explanation of statement of the conjecture with examples) [13]
- Bounds for the Erdős–Rado Sunflower Problem [2, Sections 1 and 2] or [14] or [17]

References

- [1] N. Alon and J. H. Spencer, *The probabilistic method*, Fourth edition, Wiley Series in Discrete Mathematics and Optimization, 2016.
- [2] R. Alweiss, S. Lovett, K. Wu, and J. Zhang, *Improved bounds for the sunflower lemma*, *Annals of Mathematics* **194** (2021), 795–815, preprint version available under <https://arxiv.org/pdf/1908.08483.pdf>
- [3] P. Balister, B. Bollobás, M. Campos, S. Griffiths, E. Hurley, R. Morris, J. Sahasrabudhe, and M. Tiba, *Upper bounds for multicolour Ramsey numbers*, preprint, available under <https://arxiv.org/pdf/2410.17197.pdf>
- [4] P. Balister, B. Bollobás, R. Morris, J. Sahasrabudhe, and M. Tiba, *Erdős Covering Systems*, *Acta Mathematica Hungarica* **161** (2020), 540–549, preprint version available under <https://arxiv.org/pdf/2211.01417.pdf>
- [5] P. Bennett and T. Bohman, *A natural barrier in random greedy hypergraph matching*, *Combinatorics, Probability and Computing* **28** (2019), 816–825, preprint version available under <https://arxiv.org/pdf/1210.3581.pdf>
- [6] P. Bennett and A. Dudek, *A gentle introduction to the differential equation method and dynamic concentration*, *Discrete Mathematics* **345** (2022), 17 pp., preprint version available under <https://arxiv.org/pdf/2007.01994.pdf>

- [7] J. Fox and B. Sudakov, *Dependent random choice*, Random Structures & Algorithms **38** (2011), 68–99, preprint version available under <https://arxiv.org/pdf/0909.3271.pdf>
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- [9] D. Galvin, *Three tutorial lectures on entropy and counting*, available under <https://arxiv.org/pdf/1406.7872.pdf>
- [10] J. Gilmer, *A constant lower bound for the union-closed sets conjecture*, preprint, available under <https://arxiv.org/pdf/2211.09055.pdf>
- [11] A. Heckel, *Non-concentration of the chromatic number of a random graph*, Journal of the American Mathematical Society **34** (2021), 245–260, preprint version available under <https://arxiv.org/pdf/1906.11808.pdf>
- [12] S. Janson, T. Łuczak, and A. Ruciński, *Random graphs*, Wiley Series in Discrete Mathematics and Optimization, 2011.
- [13] J. Park and H. T. Pham, *A Proof of the Kahn–Kalai Conjecture*, Journal of the American Mathematical Society **37** (2024), 235–243, preprint version available under <https://arxiv.org/pdf/2203.17207.pdf>
- [14] A. Rao, *Coding for Sunflowers*, Discrete Analysis 2020:2, 8 pp., available under <https://arxiv.org/pdf/1909.04774.pdf>
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- [16] W. Sawin, *An improved lower bound for the union-closed set conjecture*, preprint, available under <https://arxiv.org/pdf/2211.11504.pdf>
- [17] T. Tao, *The sunflower lemma via Shannon entropy*, blog post, <https://terrytao.wordpress.com/2020/07/20/the-sunflower-lemma-via-shannon-entropy/>
- [18] L. Warnke, *On Wormald’s differential equation method*, Combinatorics, Probability and Computing, to appear, preprint version available under <https://arxiv.org/pdf/1905.08928.pdf>