S4F1 Graduate Seminar on Probability Theory
Probabilistic Combinatorics

Winter Semester 2023/24

Time and location: **Fridays 12-14** in seminar room 0.006
Prerequisites: basic knowledge of probability theory, on the level of an introductory course
Lecturer: Lisa Sauermann `sauermann@iam.uni-bonn.de`

The topic of this graduate seminar is "Probabilistic Combinatorics", the study of combinatorial problems using methods from probability theory. The talks in this seminar will cover different results in this area, and many of the talks are on recent research papers. As a prerequisite, students should be familiar with basic probability theory, on the level of an introductory course. Relevant concepts from combinatorics will be introduced during the seminar (prior knowledge of basic combinatorial concepts such as sets and graphs is helpful, but not required). No prior exposure to probabilistic combinatorics is expected. The seminar may also be suitable for advanced undergraduate students interested in probability theory or combinatorics (or both).

The following is a list of possible talk topics (depending on the number of students, not all topics will be covered). Unless a specific date is noted, the dates are somewhat flexible (the order of of the talks can be changed to some extent):

- October 13. Preliminary meeting (logistical information and sign up for talks)
- October 20. Basic graph theoretic notions, proof of existence of high-girth graphs with large chromatic number, asymptotic notation, Chernoff bound (with proof)
  [7, Section 1.2 until top of page 10], [1, "Probabilistic Lens" after Chapter 3 on p. 43–44], [7, Section 2.1 until Theorem 2.8]
- October 27. Ramsey Numbers: Sketch of inductive proof of simple upper bound, probabilistic proof of lower bounds for Ramsey numbers for two colors, and for more colors
  [1, Section 1.1], [11, Section 1], [notes from previous course of lecturer]
- Entropy and applications
  [1, Section 15.7], [4, Sections 2.2, 2.3, and 3.1]
- Constant lower bound for the Union-Closed Conjecture [5]
- Random graphs: containment of small subgraphs and clique number
  [1, Sections 4.3 to 4.5]
- Proof of the Kahn–Kalai Conjecture (and explanation of statement of the conjecture with examples) [9]
- Bounds for the Erdős–Rado Sunflower Problem
  [2, Sections 1 and 2] or [10] or [12]
- Martingales and applications, in particular to the chromatic number of a random graph
  [1, Setions 7.1 to 7.4]
- Anticoncentration, the Erdős–Littlewood–Offord theorem, and applications to random matrix theory
  [notes for talk preparation provided by lecturer]
- Non-concentration of the chromatic number of a random graph [6]
- Hyperplane collections slicing all edges of the hypercube [8]
- Erdős Covering Systems [3]

The preliminary meeting for this seminar will be during the first week of classes, on Friday October 13 (at 12-14). If you are interested in giving one of the first two talks (on October 20 or October 27), feel free to email the instructor before the start of the semester. The remaining talks will be assigned at the preliminary meeting during the first week of classes.

Every student is required to meet with the instructor at least one week in advance of the talk to discuss the student's talk outline (the student is required to bring detailed notes to the meeting).

When giving a talk as a group, every student in the group is still responsible for understanding the entire topic (i.e. all of the talk content presented by the whole group) and answering questions about the topic.

The seminar will not meet on November 17 and January 12.

References

[1] N. Alon and J. H. Spencer, *The probabilistic method*, Fourth edition, Wiley Series in Discrete Mathematics and Optimization, 2016.

[2] R. Alweiss, S. Lovett, K. Wu, and J. Zhang, *Improved bounds for the sunflower lemma*, Annals of Mathematics **194** (2021), 795–815, preprint version available under `https://arxiv.org/pdf/1908.08483.pdf`

[3] P. Balister, B. Bollobás, R. Morris, J. Sahasrabudhe, and M. Tiba, *Erdős Covering Systems*, Acta Mathematica Hungarica **161** (2020), 540–549, preprint version available under `https://arxiv.org/pdf/2211.01417.pdf`

[4] D. Galvin, *Three tutorial lectures on entropy and counting*, available under `https://arxiv.org/pdf/1406.7872.pdf`

[5] J. Gilmer, *A constant lower bound for the union-closed sets conjecture*, preprint, available under `https://arxiv.org/pdf/2211.09055.pdf`

[6] A. Heckel, *Non-concentration of the chromatic number of a random graph*, Journal of the American Mathematical Society **34** (2021), 245–260, preprint version available under `https://arxiv.org/pdf/1906.11808.pdf`

[7] S. Janson, T. Łuczak, and A. Ruciński, *Random graphs*, Wiley Series in Discrete Mathematics and Optimization, 2011.

[8] O. Klein, *Slicing all Edges of an n-cube Requires $n^{2/3}$ Hyperplanes*, Proceedings of IEEE Symposium on Foundations of Computer Science 2023, preprint version available under `https://arxiv.org/pdf/2212.03328.pdf`

[9] J. Park and H. T. Pham, *A Proof of the Kahn–Kalai Conjecture*, Journal of the American Mathematical Society, to appear, preprint version available under `https://arxiv.org/pdf/2203.17207.pdf`

[10] A. Rao, *Coding for Sunflowers*, Discrete Analysis 2020:2, 8 pp., available under `https://arxiv.org/pdf/1909.04774.pdf`

[11] W. Sawin, *An improved lower bound for multicolor Ramsey numbers and a problem of Erdős*, Journal of Combinatorial Theory Series A **188** (2022), Article ID 105579, 11 p., preprint version available under `https://arxiv.org/pdf/2105.08850.pdf`

[12] T. Tao, *The sunflower lemma via Shannon entropy*, blog post, `https://terrytao.wordpress.com/2020/07/20/the-sunflower-lemma-via-shannon-entropy/`