On the mathematical theory of Landau Damping

Graduate Seminar on Analysis (S4B1)

Winter Term 2017

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Synopsis

Landau damping, named after its discoverer L.D. Landau, is a phenomena observed in plasma wherein there is an exponential decay in the oscillations of the number density of electrons and so stability is achieved in some area of the phase-space. The damping of such collisionless oscillations was predicted by Landau in 1946 who deduced this effect from a mathematical study without reference to a physical explanation.

The starting point in the investigation of this phenomena is the Vlasov-Poisson equation

$$\partial_t f + v \cdot \nabla_x f + F(t,x) \cdot \nabla_v f = 0 \tag{1}$$
$$F(t,x) = -\nabla W *_x \rho, \quad \rho = \int f(t,x,v) dv$$

which is a time-reversible transport equation and it determines the statistical properties of plasmas. Here $f = f(t,x,v)$ is a time-dependent density distribution in phase space (position, velocity) and W is the Coulomb potential.

Laudau studied a linearised version of this model and concluded that the electrical forces weakened spontaneously over time without a corresponding increase in entropy. The problem which remained was whether or not Landau's results also apply to the nonlinear model.

We will start with the analysis of the Vlasov-Poisson equation (1) and its qualitative properties and we will focus on the mathematical theory, developed by Cédric Villani and Clément Mouhot, which allowed to fill this gap and demonstrate that relaxation is possible in confined reversible systems, without entropy increase.

Prerequisites: basic knowledge of PDEs and functional analysis is essential.

Organization: A preliminary meeting will take place on Wednesday 26.07.17 at 14.15am in room 2.025.
Interested students are asked to contact us by email:
velazquez@iam.uni-bonn.de; nota@iam.uni-bonn.de

Literature sample:

1. Villani, C.: Landau damping, Lecture notes
http://smai.emath.fr/cemracs/cemracs10/PROJ/Villani-lectures.pdf

For further readings:

2. Mouhot, C. and Villani, C.: On Landau damping, *Acta Mathematica*, 207 (1), pp. 29–201 (2011)

Curiosity: there is a book on the topic written for broad audience, i.e.

3. Villani, C.: Birth of a Theorem: A Mathematical Adventure.
The Bodley Head, London (2015)