Seminar on Thin-film Flows with Non-trivial Dynamics (S4B1)

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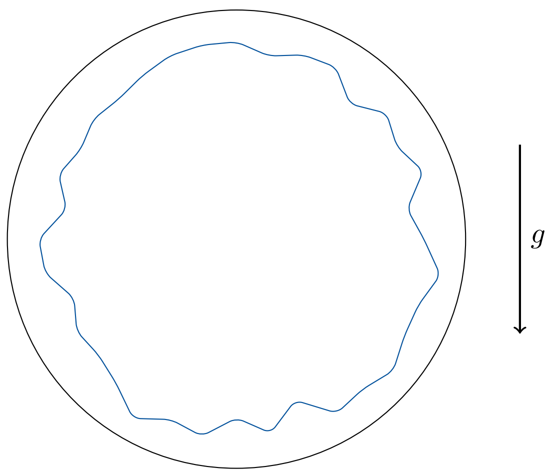
Date and Place of the Seminar: Mondays, 14–16 (online)

Abstract. Rimming flows are flows of viscous fluid layers on the inside of a horizontally rotating cylinder. The combined effect of gravitational forces and fluid rotation results in the onset of a rich class of patterns which include different types of steady states, oscillatory patterns, shock like structures and also chaotic behaviours. The patterns depend strongly on different parameters of the problem, which include inertia, gravity, speed of rotation of the cylinder, surface tension at the interfaces, and the viscosity of the fluid among others.



The models used to describe rimming flows are free boundary problems emerging from the governing Euler or (Navier)-Stokes equations. However, in this seminar we restrict our analysis to the case of 'thin' fluid layers. This allows us to use the so-called lubrication approximation [1,2], in which we consider the limit of a vanishing film height to derive from the Euler- or Navier–Stokes systems an evolution equation for the film height. The resulting equations are similar to the classical thin-film equation. A typical equation is

$$h_t + \Big(\big(h - \tfrac{1}{3}\cos(\theta)h^3\big) + h^3\left(h_\theta + h_{\theta\theta\theta}\right) + \sin(\theta)h^3 h_\theta\Big)_\theta = 0, \quad t > 0,\ \theta \in S^1. \tag{1}$$

Since (1) is an equation of fourth order, no maximum or comparison principles are available. The flux terms account for the rotation of the cylinder, the viscous forces, the surface tension and the gravitational forces, respectively. Depending on a subtle interplay of these different physical quantities, solutions may develop a rather complicated asymptotic behaviour that has been barely

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studied from the mathematical point of view. The linearisation of (1) near stationary solutions [3, 4] results in operators with unusual spectral properties, due to the fact that these operators are neither symmetric nor skew-symmetric.

The focus of this seminar is to understand the derivation of different equations modelling rimming flows that can be found in the literature. Moreover, we aim to understand the influence of the different terms/ forces on the asymptotic behaviour of solutions.

PREREQUISITES. Basic notions of functional analysis and partial differential equations are required, while no previous knowledge on fluid dynamics is needed.

LITERATURE.

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(2) V. V. Pukhnachev. *Motion of a liquid film on the surface of a rotating cylinder in a gravitational field*, Journal of Applied Mechanics and Technical Physics, 1977.

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(5) S. B. G. O'Brien and E. G. Gath. *The location of a shock in rimming flow*, Physics of fluids **10**. 1998.

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(7) M. Chugunova and S. Pyatkov. *Compactly supported solutions for a rimming flow model*, Nonlinearity, 2014.

(8) E. S. Benilov and S. B. G. O'Brien. *Inertial instability of a liquid film inside a rotating horizontal cylinder*, Physics of fluids **17**, 2005.

(9) E. S. Benilov, N. Kopteva and S. B. G. O'Brien. *Does surface tension stabilize liquid films inside a rotating horizontal cylinders ?*, Q. J Mech. Appl. Math **58**, 2005.

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An overview on the physics of the rimming flows can be found in the review paper:

- G. Seiden and P. J. Thomas. *Complexity, segregation, and pattern formation in rotating-drum flows*, Rev. Mod. Phys. **83**, 2011.