Seminar on Fluid Dynamics (S4B2)

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Preliminary meeting: March 01, 2021, 14:00 (via Zoom)

Abstract. Fluid dynamical problems appear in a variety of applications in chemistry, physics and engineering and their mathematical modelling and analysis is therefore a rather active field of applied analysis.

The seminar covers two main research areas in fluid dynamics. In the first part we are concerned with existence, uniqueness and qualitative properties of weak solutions to the *Navier–Stokes equations*

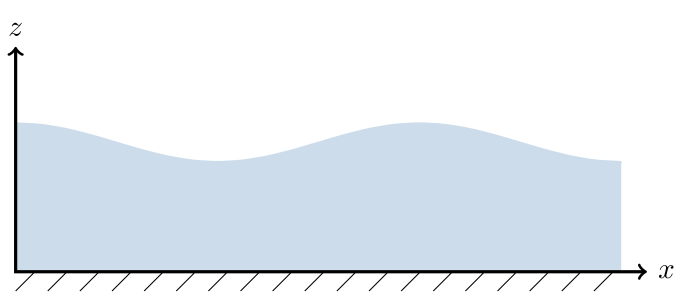
$$\begin{cases} u_t + \big(u \cdot \nabla u\big) = \nabla \cdot \big(-pI + \mu \nabla u\big), \quad t > 0, x \in \mathbb{R}^d \\ \nabla \cdot u = 0, \quad t > 0, x \in \mathbb{R}^d, \end{cases} \tag{1}$$

describing the dynamics of a viscous fluid on a solid bottom. While in dimension $d = 2$ global existence, uniqueness and regularity properties are known, one of the main open problems in fluid dynamics is the question on existence of smooth solutions when $d = 3$. This is in fact one of the Clay Foundation's Millennium Problems.

Thin fluid films are ubiquitous in nature, science and technology. In the second part of the seminar, we study the dynamic behaviour of these fluid films, which may be described by so-called *thin-film equations*

$$u_t + \nabla \cdot \big(|u|^n \nabla \Delta u\big) = 0, \quad t > 0, x \in \Omega. \tag{2}$$

Equations of the form (2) may be derived from the Navier–Stokes system (1) in the asymptotic limit of a vanishing film height. A typical application is the modelling of droplets on inclined planes. Since (2) is an equation of fourth order, no maximum or comparison principles are available such that the corresponding solution theory may differ much from the one for second-order analogues such as the porous-medium equation. A further important property of (2) is that the equation degenerates when the film height u tends to zero. Talks may cover questions regarding existence and uniqueness of weak solutions and their behaviour near points where u vanishes.



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Prerequisites. Basic notions of functional analysis and partial differential equations are required, while no previous knowledge on fluid dynamics is needed.

There is no overlap with the seminar *Models arising in geophysical fluid dynamics* (S4B1) which can rather serve as a supplementary course.

Literature.

(1) F. Bernis, A. Friedman. *Higher Order Nonlinear Degenerate Parabolic Equations*, J. Diff. Equ., 1990.

(2) J. Leray. *Sur le mouvement d'un liquide visqueux emplissant l'espace*, Acta Math., 1934.

(3) E. Wiedemann. *Weak-strong uniqueness in fluid dynamics*, arXiv:1705.04220, 2017.