# Spectral theory and random differential operators

Lectures Tuesdays 10(c.t.)–12 Room 0.008 (Endenicher Allee 60)

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Random differential operators are used in the modelling of physical processes in disordered, unknown, or uncertain media. Related elegant theories of Anderson localization, integrated density of states (IDS), and stochastic homogenization have been developed intensively, but still contain many long-standing open problems. These theories are mainly connected with random selfadjoint operators and conservative systems, whereas some of resulting spectral properties are deterministic.

The mathematical physics theory of non-Hermitian matrices, which was developed to model spectral properties of open or dissipative systems [FS15], has evolved recently in directions of stochastic resonances associated with random PDEs [S14, AK21, K24] and random difference equations [K16]. These directions concern either continuation resonances of random structures surrounded by a homogeneous medium and equipped with radiation condition at $\infty$, or models where the stochastic surrounding is replaced by a random boundary condition [K24]. The associated random spectra are usually non-real, non-deterministic, and can be often represented by (stochastic) point proccesses in $\mathbb{C}$.

The course is primarily aimed on random spectral theory for nonselfadjoint differential and difference operators. It is planned to introduce definitions and basic facts concerning

- point processes [LP17], random operators [S, DaZa],
- spectral theory of selfadjoint and nonselfajdoin operators [AG, K],

as well as

- elements of modern spectral theory like resonances and boundary tuples [DyZw, K24].

After that we concentrate our attention on several specific stochastic models involving

- random continuation resonances [K16, AK21],
- random conservative and dissipative boundary conditions for PDEs [K24],
- and various point processes that generate random coefficients or describe random spectra.

Whenever time allows us, we will compare these new random dissipative models with classical and modern results of the well-developed selfadjoint theories of Anderson localization, IDS, and stochastic homogenization.

**Prerequisites: basic PDEs, basic Functional Analysis, basic Probability** (Lebesgue-spaces, linear PDEs, Banach spaces, Hilbert spaces, scalar-valued random variables).

**Basic knowledge of the following topics may be useful, but not necessary**:
basic understanding of stochastic point proccesses and Sobolev spaces.

## Literature


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