Winter Semester 2025/26
S5B5-Graduate Seminar on Advanced Topics in Functional Analysis & Operator Theory

Random operators and resonances

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Preliminary meeting: 09 July 2025, 14:15, room N0.008 (Neubau).

If you cannot come to the preliminary meeting, but are interested in the seminar, please, feel free to contact the organizers by the email given above.

Random difference and differential operators are used to model physical processes in disordered or uncertain media. The theories involving the Anderson model for selfadjoint discrete Schrödinger operators, integrated densities of states, and Lyapunov exponents, and the multiscale analysis have been developed intensively, but still contain long-standing open problems. These theories are mainly connected with random selfadjoint operators and conservative systems.

The theory of non-Hermitian random matrices, which originated in Mathematical Physics and is aimed on spectral properties of open (non-conservative) systems, has evolved recently in the direction of stochastic resonances associated with random PDEs and difference equations. Related random spectra are usually non-real, non-deterministic, and can be often represented by (stochastic) point proccesses in the complex plane.

The seminar is aimed on the following well-developed topics of Random Spectral Theory for the selfadjoint Anderson model, as well as on connections of these classical topics with relatively new non-selfadjoint random spectral problems:

- random discrete Schrödinger operators (the Anderson model) [K];
- ergodic operators and their spectra [K];
- the density of states [K];
- the Lyapunov exponent for the 1-d Anderson model and the Thouless formula [CFKS, PF];
- continuation resonances for the 1-d Anderson model [K16].

While the seminar is closely connected with seminars and courses of previous semesters on the Anderson localization, random matrices, point processes, and spectral theory, the selection of themes is somewhat different and is aimed to complement the earlier seminars and courses on random operators.

Prerequisites: Basic Functional Analysis, basic Spectral Theory in Hilbert spaces, basic Probability

Basic knowledge of the following topics may be useful, but not necessary: discrete Laplacian, discrete Anderson model, continuation resonances, point processes.

[CFKS] Cycon, H.L., Froese, R.G., Kirsch, W. and Simon, B., 1987. Schrödinger operators with applications to quantum mechanics and global geometry, Springer.

[K] Kirsch, W., 2008. An invitation to random Schrödinger operators. With an appendix by F. Klopp. In: Random Schrödinger operators, pp. 1–119, Soc. Math. France, Paris, see also the arXiv preprint arXiv:0709.3707; https://doi.org/10.48550/arXiv.0709.3707

[K16] Klopp, F., 2016. Resonances for large one-dimensional "ergodic" systems, Analysis & PDE 9(2), 259–352. http://dx.doi.org/10.2140/apde.2016.9.259

[PF] Pastur, L.A. and Figotin, A., 1992. Spectra of random and almost-periodic operators. Berlin: Springer.