Summer Semester 2024 S4B2 - Graduate Seminar on PDE

Dissipative boundary value problems for wave equations

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Preliminary meeting: **07 February 2024, 12:00,** room 0.011 (Endenicher Allee 60).

If you cannot come to the preliminary meeting, but are interested in the seminar, please, feel free to contact the organizers by the email given above.

The seminar is aimed to Spectral Theory of unbounded selfadjoint and dissipative operators with examples stemming from wave equations.

While the spectral theory of operators that are selfadjoint and/or compact is a part of the courses of Functional Analysis, PDEs, and Mathematical Physics, it is not easy to cover in the wide scope of these fundamental courses many basic topics involving unbounded linear operators. A list of such basic, but more advanced, operator theory topics may include

- Friedrichs extensions and representation theorems for sesquilinear forms,
- partial differential operators with essential and/or nonreal spectra,
- contraction semigroups and their connections with m-dissipative and sectorial operators,
- PDEs with boundary conditions more sophisticated than the simplest Dirichlet/Neumann cases, e.g., the "3rd boundary condition", impedance/absorbing boundary conditions, radiation conditions at ∞,
- continuation resonances/scattering poles.

Many of these topics are needed as a background for nonlinear PDEs, Optimization, Bifurcation Theory, and Random Spectral Theory, as well as for Physics applications, e.g., in Quantum Mechanics, Photonics, or Kinetic Theory.

The goal of the seminar is to cover the basics topics of Spectral Theory mentioned above and to consider their applications to various selfadjoint/dissipative boundary value problems for acoustic and electromagnetic wave equations.

Prerequisites: Basic PDEs, basic Functional Analysis (Lebesgue-spaces, linear PDEs, Hilbert spaces, bounded and compact operators in Hilbert spaces).

Basic knowledge of the following topics may be useful, but not necessary: Sobolev spaces, Banach spaces, evolution equations in Hilbert or Banach spaces, (linear) operator semigroups.

Literature

[1] Borthwick, D., Spectral theory: Basic concepts and applications. Springer Nature, 2020.

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[3] Dyatlov, S. and Zworski, M., Mathematical theory of scattering resonances. AMS, 2019.

[4] Eller, M. and Karabash, I.M., 2022. M-dissipative boundary conditions and boundary tuples for Maxwell operators. Journal of Differential Equations 325, pp.82-118.

[5] Gorbachuk, V.I. and Gorbachuk, M.L., Boundary Value Problems for Operator Differential Equations, Springer Science & Business Media, 1991.

[6] Kato, T., Perturbation theory for linear operators. Springer-Verlag, 1976.

[7] Leis, R., Initial Boundary Value Problems in Mathematical Physics. Tubner, 1986.