Winter Semester 2023/24
V5B6 - Selected Topics in Analysis and Calculus of Variations

Homogenization-convergence and optimization of eigenvalues

Lectures Thursdays 12(c.t.)–14 **Room 0.011** (Endenicher Allee 60)

Illia M. Karabash e-mail for questions:<ikarabas@uni-bonn.de>

The H-convergence and the G-convergence are two types of homogenization-convergences that are used to address the existence of optimizers in the problems of Structural Optimization. The course is particularly aimed to the application of these convergences to optimization of eigenvalues associated with wave equations in structured media, e.g., in the context of Photonics. The main goals are:

- Various definitions of H-/G-convergence and connections between these definitions.
- Optimization of eigenvalues of selfadjoint partial differential operators and systems.
- The G-closure problem and modeling of composite materials.
- Wave equations, Photonics, and the associated spectral theory.

If there is time, recent developments in optimization of eigenvalues of nonselfadjoint differential operators and resonances will be discussed, as well as connections with Γ-convergence.

Prerequisites: Basic PDE, basic Functional Analysis (Lebesgue-spaces, linear PDEs, Banach spaces, strong/weak/weak-* convergences).

Basic knowledge of the following topics may be useful, but not necessary: Basic understanding of Sobolev spaces and operators with discrete spectra in Hilbert spaces.

Literature

[A] Allaire, G., Shape Optimization by the Homogenization Method. Springer, 2002.

[ATL89] Alvino, A., Trombetti, G. and Lions, P.L., 1989. On optimization problems with prescribed rearrangements. Nonlinear Analysis: Theory, Methods & Applications, 13(2), pp.185-220.

[ACL] Assous, F., Ciarlet, P. and Labrunie, S., Mathematical foundations of computational electromagnetism. New York: Springer International Publishing, 2018.

[CK] Cherkaev, A., and Kohn, R., Eds., Topics in the mathematical modelling of composite materials. Boston: Birkhäuser, 1997.

[CL96] Cox, S., and Lipton, R., 1996. Extremal eigenvalue problems for two-phase conductors. Archive for Rational Mechanics and Analysis 136(2), pp.101–118.

[H] Henrot, A., Extremum problems for eigenvalues of elliptic operators. Springer Science & Business Media, 2006.

[KKV20] Karabash, I.M., Koch, H., and Verbytskyi, I.V., 2020. Pareto optimization of resonances and minimum-time control. Journal de Mathématiques Pures et Appliquées 136, pp.313–355.

[NKT08] Notomi, M., Kuramochi, E. and Taniyama, H., 2008. Ultrahigh-Q nanocavity with 1D photonic gap. Optics Express, 16(15), pp.11095-11102.

[VS21] Vasco, J.P. and Savona, V., 2021. Global optimization of an encapsulated Si/SiO 2 L3 cavity with a 43 million quality factor. Scientific Reports 11(1), p.10121.