Stochastic Analysis – Problem Sheet 6.

Tutorial classes: Mon June 6th in SemR 0.008. Philipp Boos <s6phboos@uni-bonn.de>. Solutions will be collected at the beginning of the tutorial session. At most in groups of 3.

**Exercise 1.** Prove directly that the $h$-transform gives a transformation of martingale problems from $\text{MP}(x_0, b, a)$ to $\text{MP}(x_0, \tilde{b}, a)$ where $\tilde{b} = b + a \nabla (\log h) \nabla$. (That is, reproduce the argument of the notes without relying on the Itô decomposition of the process)

**Exercise 2.** Let $(X, \mathbb{P})$ be a solution of the Martingale Problem $\text{MP}(x_0, b, a)$. Generalise appropriately the Girsanov transform to construct a measure $\mathbb{Q}$ under which the process $X$ solves a martingale problem with a different drift. For simplicity, assume that all the necessary integrability conditions are satisfied. (Who takes the place of the Brownian motion?)

**Exercise 3.** Use Girsanov transform to prove that the weak solution of the SDE

$$dX_t = b_t(X)dt + dB_t$$

where $b: \mathbb{R}_+ \times C(\mathbb{R}_+; \mathbb{R}^d) \to \mathbb{R}^d$ is a bounded, previsible drift, is unique in law.