
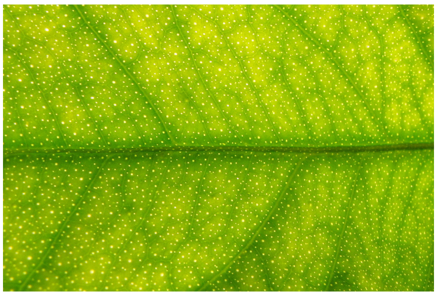
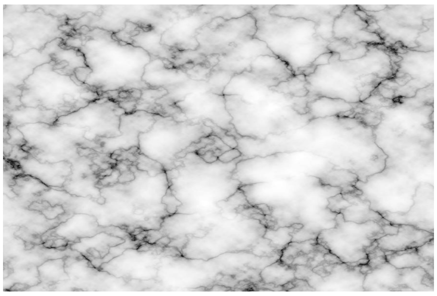

Selected Topics in Analysis and PDEs:
Stochastic Homogenization
Peter Gladbach – Summer 2020

Content

In this class, we will study the macroscopic behavior of solutions to the elliptic Dirichlet problem for $\Omega \subset \mathbb{R}^d$, $u_\varepsilon : \Omega \to \mathbb{R}$,

$$\begin{cases} \operatorname{div}\left(a\left(\frac{x}{\varepsilon}\right)\nabla u_\varepsilon(x)\right) = f(x) & \text{in } \Omega \\ u_\varepsilon(x) = g(x) & \text{on } \partial\Omega \end{cases} \tag{1}$$

as $\varepsilon \to 0$, where the material coefficient $a\left(\frac{x}{\varepsilon}\right)$ oscillates rapidly, as in biological tissues and composite materials.

Examples are:

- The equilibrium distribution of an enzyme inside an inhomogeneous biological tissue.
- The equilibrium temperature in a fibrous material.
- The electric potential in a composite resistor.

To this end, we will identify a homogenized elliptic Dirichlet problem

$$\begin{cases} \operatorname{div}\left(\overline{A}\nabla \overline{u}(x)\right) = f(x) & \text{in } \Omega \\ \overline{u}(x) = g(x) & \text{on } \partial\Omega, \end{cases} \tag{2}$$

such that $u_\varepsilon \to \overline{u}$. The homogenized material coefficient should be independent of the position and deterministic, even if $a\left(\frac{x}{\varepsilon}\right)$ is a random variable.

We will first treat the periodic case $a(x) = a(x+z)$ for $z \in \mathbb{Z}^d$ and then move on to the stochastic case where $a(x,\omega)$ is a stationary coefficient field. Here we will prove the ergodic theorem and show convergence of the solutions to (1) to solutions of (2). Finally, we will show an upper bound for the convergence rate.

Topics

- Periodic homogenization for the elliptic problem
- Stochastic homogenization for the elliptic problem
- Quantitative stochastic homogenization

Prerequisites

- Sobolev spaces
- Introduction to PDEs

References

[1] Scott Armstrong, Tuomo Kuusi, and Jean-Christophe Mourrat, *Quantitative stochastic homogenization and large-scale regularity*, arXiv preprint, 2017, arXiv.org/abs/1705.05300.

[2] Stefan Neukamm, *An introduction to the qualitative and quantitative theory of homogenization*, lecture notes, 2017, arxiv.org/abs/1707.08992.