V5B3 Adv. Top. in PDE & Math. Mod, SS 2025

Semiclassical methods in statistical and quantum mechanics.

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Description. Many problems in equilibrium statistical mechanics and quantum mechanics can be reformulated as measures on $\mathbb{R}^N$ of the form

$$\frac{1}{Z_N} e^{-\beta S(\varphi_1,\ldots,\varphi_N)} \prod_{j=1}^{N} d\varphi_j,$$

where the function S encodes the physical information on the model and grows fast enough to ensure that $e^{-\beta S} \in L^1(\mathbb{R}^N)$, $d\varphi_j$ is the Lebesgue measure on $\mathbb{R}$, Z_N is a normalization constant, and $\beta > 0$ may correspond to the inverse temperature $\beta = \frac{1}{T}$ or the inverse Plank constant $\beta = \frac{1}{h}$. In some cases, S may be also complex-valued. This measure contains two parameters β, N which generate two important limits.

The limit $\beta \to \infty$ (equivalently $h \to 0$ or $T \to 0$) corresponds to the *semiclassical limit* (or low temperature limit).

The limit $N \to \infty$ corresponds to the *thermodynamic limit*, where the number of particles diverges.

The goal of this lecture is to introduce some tools to study these two limits and their mixture. We will mostly focus on two powerful tools:

- the transfer operator approach, which reduces the problem to the analysis of high powers of a compact operator on a one-particle Hilbert space.
- the Witten-Laplacian approach which allows to compare the measure (precisely certain correlation functions) with a Gaussian measure, as long the function S is convex.

We will mostly follow the book

Semiclassical analysis, Witten Laplacians and statistical mechanics,

by B. Helffer (World Scientific).

Prerequisites. Functional analysis. Some basic knowledge in probability/physics may be useful but is not necessary.