Topics in Mathematical Quantum Mechanics

Graduate Seminar (S4B2)

Winter Term 2019/2020

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Topic One of the most fundamental problems of quantum mechanics is the question whether it describes systems where matter is stable, i.e., has stability properties as we observe in our world. In particular stability requires that the energy is bounded below and the minimal energy should scale linearly in the number of particles (implying that the volume of a sample is at least proportional to the number of particles it contains).

It was long known that classical mechanics predicts unstable matter. Therefore the proof of stability in quantum mechanics in the 1960's was a huge success. Moreover, the proof revealed that besides the quantum nature also the Pauli exclusion principle is necessary to obtain stability.

In this seminar we will discuss a modern proof of this result. In mathematical terms we need a good control of the smallest eigenvalue of the many-body Hamiltonian $\mathcal{H}$ for the electrons. Here $\mathcal{H}$ is a self-adjoint operator that acts on $\psi \in L^2(\mathbb{R}^{3N})$ by

$$\mathcal{H}\psi\,(x_1,\ldots,x_N) = \sum_{i=1}^{N}\Big(-\Delta_{x_i} + V_{\text{ext}}(x_i)\Big)\psi(x_1,\ldots,x_N) - \sum_{i\neq j}\frac{1}{|x_i-x_j|}\psi(x_1,\ldots,x_N)$$

where $V_{\text{ext}}:\mathbb{R}^3\to\mathbb{R}$ is an electric potential generated by the atomic nuclei. The most important ingredient in our proof are Lieb-Thirring inequalities which state that

$$\sum_{\lambda_j<0}|\lambda_j|^\gamma \le C_{\gamma,n}\int_{\mathbb{R}^n}|V(x)|^{\gamma+\frac{n}{2}}\,\mathrm{d}x$$

for $\gamma\ge 0$ and $n\ge 3$ where $V:\mathbb{R}^n\to\mathbb{R}$ is a potential and λ_j denote the negative eigenvalues of the operator $-\Delta+V$. Moreover, we need to investigate how well we can control the energy of the system if we only know its one particle density ρ_ψ. This is a question of independent interest in particular in the context of quantum chemistry.

In case of interest, we might also discuss some mathematical results for the energy Bose-Einstein condensates and their dynamics in the last part of the seminar.

Prerequisites Knowledge of measure theory, functional analysis and partial differential equations (e.g. participation in the Bachelor course 'Functional analysis and partial differential equations')

Preliminary meeting **Tuesday, October 8 at 12:15 in room 1.008**

Time and Location Tuesdays, 12:15-13:45, room 1.008

Further information Contact us by email: buchholz@iam.uni-bonn.de
Also feel free to contact us if you would like to attend but you have a scheduling conflict.

References

[1] Elliott H. Lieb and Robert Seiringer. *The stability of matter in quantum mechanics.* Cambridge University Press, Cambridge, 2010.

[2] Elliott H. Lieb, Robert Seiringer, Jan Philip Solovej, and Jakob Yngvason. *The mathematics of the Bose gas and its condensation*, volume 34 of *Oberwolfach Seminars*. Birkhäuser Verlag, Basel, 2005.