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Inelastic collapse of three particles in dimension $d \geq 2$

Théophile Dolmaire
dolmaire@iam.uni-bonn.de

The Boltzmann equation can be derived rigorously from a system of elastic hard spheres (Lanford's theorem). In the case of large systems of particles that interact inelastically (sand, snow, interstellar dust), the derivation of the inelastic Boltzmann equation is still open. One major difficulty, already at the microscopic level, comes from the phenomenon of inelastic collapse, when infinitely many collisions take place in finite time.

Assuming that the restitution coefficient r is constant, we obtain general results of convergence and asymptotics concerning the variables of the dynamical system describing a collapsing system of particles. We prove a complete classification of the singularities when a collapse of three particles takes place, obtaining only two possible orders of collisions between the particles: either the particles arrange in a nearly-linear chain (studied in [4]), or they form a triangle, and we show that, after sufficiently many collisions, the particles collide according to a unique order of collisions, which is periodic. Finally, we construct explicit initial configurations leading to a nearly-linear collapse in a stable way, such that the angle between the particles at the time of collapse can be chosen a priori, with an arbitrary precision.

Besides, considering a different law of collision, prescribing that a fixed quantity of kinetic energy is lost during each collision, we prove that the flow of such a system of particles conserves the measure in the phase space, whereas the kinetic energy is not conserved. From these results, we deduce an Alexander's theorem for such systems of particles: for almost every initial datum, the dynamics of such systems is globally well-posed.
The results are taken from [1], [3], [2], obtained in collaboration with Juan J. L. Velazquez (Universität Bonn).

References

[1] Théophile Dolmaire, Juan J. L. Velázquez, "Collapse of inelastic hard spheres in dimension $d \geq 2$", preprint arXiv:2402.13803v2 (02/2024).

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[3] Théophile Dolmaire, Juan J. L. Velázquez, "Properties of some dynamical systems for three collapsing inelastic particles", preprint arXiv:2403.16905 (03/2024).

[4] Tong Zhou, Leo P. Kadanoff, "Inelastic collapse of three particles", *Physical Review E*, **54**:1, 623–628 (07/1996).