Wave turbulence and kinetic theory

Graduate Seminar on Analysis (S4B1)

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Winter Semester 23/24

Possible Topics

- Introductory talk: Introduction to coagulation-fragmentation equations (Iulia Cristian)
- **Gelation in coagulation-fragmentation equations**

  Ideally, the mass of the coagulating system is preserved in time. However, it can be shown that if particles interact too quickly, some mass may be lost, even instantaneously. This phenomenon is directly linked to the **formation of a gel** in the case of polymerization. We will discuss specific examples available in the literature in which mass loss occurs.

  – Possible talks

  1. Gelation in coagulation-fragmentation models [BLL20, Section 9.1]
  2. Instantaneous gelation in coagulation-fragmentation models [BLL20, Section 9.2]
  3. Gelation in coagulation models - a different proof [EMP02]

- **Stationary solutions for coagulation equations with source term**

  We also examine the scenario where additional particles are introduced into the interacting particle system from an external source. Do we have a competition between the newly injected particles and those that were lost due to previous interactions? Specifically, we will investigate whether **stationary solutions** in time exist.

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– Possible talks

1. Part I, see [FLNV21, p. 809-836]
2. Part II, see [FLNV21, finish Existence results: continuous model + Estimates and regularity]
3. Stationary solutions for coagulation equations when a source term is added in the L^1 context, see [Lau20]

- **Weak Turbulence for the Nonlinear Schrödinger Equation**

  We will also discuss the Cauchy problem for a kinetic equation arising in the weak turbulence theory for the cubic **nonlinear Schrödinger equation**. Our focus will be on exploring the qualitative properties of the solutions, such as their long-term behavior, potential for blow-up, and condensation within a finite time frame.

  – Possible talks

  1. Well-posedness results [EV15, Section 2]
  2. Qualitative behaviors of the solutions [EV15, Section 3]
  3. Solutions without condensation: Pulsating behavior [EV15, Section 4]

References

[BLL20] J. Banasiak, W. Lamb, and P. Laurençot. *Analytic Methods for Coagulation-Fragmentation Models, Volume II*. CRC Press, 2020.

[BZ98] A. M. Balk and V. E. Zakharov. Stability of weak-turbulence Kolmogorov spectra. *American Mathematical Society Translations*, 182(2):31–82, 1998.

[EMP02] M. Escobedo, S. Mischler, and B. Perthame. Gelation in coagulation and fragmentation models. *Communications in Mathematical Physics*, 231:157–188, 2002.

[EV15] M. Escobedo and J. J. L. Velázquez. On the Theory of Weak Turbulence for the Nonlinear Schrödinger Equation. *Memoirs of the American Mathematical Society*, 238(1124), 2015.

[FLNV21] M. Ferreira, J. Lukkarinen, A. Nota, and J. Velázquez. Stationary non-equilibrium solutions for coagulation systems. *Archive for Rational Mechanics and Analysis*, 240:809–875, 2021.

[Lau20] P. Laurençot. Stationary solutions to Smoluchowski's coagulation equation with source. *North-Western European Journal of Mathematics*, 6:137–164, 2020.