# Littlewood-Paley decomposition and partial differential equations

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Graduate Seminar on Analysis (S4B1)

## Winter Term 2021

## Synopsis

In recent years, arguments based on Littlewood-Paley decomposition and the paradifferential calculus by Bony [3] have proved to be very efficient for the study of various evolution equations [1].

In this seminar we will introduce the Littlewood-Paley decomposition, Besov spaces as well as the paraproduct decomposition based on the lecture notes [4], [5]. In the following we will consider various PDEs. We start with estimates for the heat and transport equation used to derive existence/uniqueness results for the incompressible Navier-Stokes equations and the Camassa-Holm equation (a shallow water equation) in Besov spaces, respectively.

Furthermore, we study the local well-posedness of the compressible Navier-Stokes equations as well as the Navier-Stokes equations with anisotropic viscosity. The latter problem uses an anisotropic version of the Littlewood-Paley decomposition [1], [4].

Depending on the number of participants we will also discuss the simplified proof of the nonlinear Landau damping problem for the Vlasov-Poisson equation by Bedrossian, Masmoudi, Mouhot [2].

**Prerequisites:**   basic knowledge of PDEs and functional analysis is essential.

**Organization:**   A preliminary meeting will take place on **Monday 12.7.2021 at 14:15** via Zoom.
Zoom-Link:
https://uni-bonn.zoom.us/j/95897065064?pwd=VHRvYzRTek1mVnBwV2I5NlQxT2VZQT09
Meeting-ID: 958 9706 5064
Passcode: 676772

## References

[1] Hajer Bahouri, Jean-Yves Chemin, and Raphaël Danchin. *Fourier Analysis and Nonlinear Partial Differential Equations*. Springer Berlin Heidelberg, 2011.

[2] Jacob Bedrossian, Nader Masmoudi, and Clément Mouhot. Landau damping: Paraproducts and Gevrey regularity. *Annals of PDE*, 2(1):4, 2016.

[3] Jean-Michel Bony. Calcul symbolique et propagation des singularités pour les équations aux dérivées partielles non linéaires. *Annales scientifiques de l'École Normale Supérieure*, 4e série, 14(2):209–246, 1981.

[4] Jean-Yves Chemin. Localization in Fourier space and Navier-Stokes system. Lecture Notes: http://citeseer.ist.psu.edu/viewdoc/summary?doi=10.1.1.458.8969.

[5] Raphaël Danchin. Fourier analysis methods for PDE's. Lecture Notes: https://perso.math.u-pem.fr/danchin.raphael/cours/courschine.pdf.

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