Non-local equations modelling physiologically structured populations

Graduate seminar on PDE (S4B2), Summer term 2023

Dr. E. Franco, Prof. Dr. J.J.L. Velázquez

Introduction

During this seminar we study a class of models, coming from biology, that aim at describing the evolution in time of a population. In this context a population is intended as a collection of individuals (for instance of cells or of humans). The individuals in the population are characterized by a feature, called individual state or structuring variable, which changes continuously in time. An example of a structuring variable is age, which changes in time due to aging, or size, which might change in time due to a growth process. The number of the individuals in the population, as well as the population distribution along the structuring variable, changes in time also due to birth-death events.

A mathematical model of a structured population consists in an equation (or a system of equations), whose solution describes the evolution in time of the population distribution along the structuring variable. The resulting equation is often a partial differential equation that includes a non-local term, which is due to the birth-death events, and a transport term, which is due to the smooth development of the structuring variable (for instance growth and aging), [4]. There are, though, other possible ways to formulate this models, for instance via integral renewal equations, [1], [3].

In this seminar we plan to study some models of populations structured by different structuring variables. We will pay particular attention to age and to size. We will see different ways to write the models and different approaches to study the resulting equations. In particular, we will study the long-term behavior of their solutions. One important tool in the study of the long-term behaviour of the solution of these equations, is the generalized relative entropy inequality, [4]. We will see how to apply this technique to some age structured and size structured models.

When, instead, the model is formulated as a renewal integral equation it is possible to use Laplace transforms methods to study the long-term behaviour, ([3], [1]). Again, we will see these techniques applied to an age structured model and to a size structured model of cell growth and fission.

Depending on the number of participants, we will also discuss an application of Doeblin's method to study an age structured population model, [2].

Prerequisites: basic notions of functional analysis and partial differential equations

Preliminary meeting: Wednesday 25th January 2023 at 14.15, Room 2.040 Endenicher Allee 60

References

[1] O. Diekmann, *The dynamics of physiologically structured populations*, Springer-Verlag, Berlin Heidelberg, 1986.

[2] P. Gabriel, *Measure solutions to the conservative renewal equation*, ESAIM: Proceedings and Surveys, 62 (2018), pp. 68–78.

[3] M. Iannelli and F. Milner, *The basic approach to age-structured population dynamics*, vol. 10, Springer, Berlin, 2017.

[4] B. Perthame, *Transport equations in biology*, Springer, Berlin, 2006.