

Semigroup theory for linear evolution equations

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In this course, we introduce the basic theory of one-parameter semigroups of bounded linear operators on Banach spaces and discuss how it is related to the stability theory of abstract evolution equations. The main emphasis is on the spectral properties as well as on the Lyapunov methods, with applications to the selected classes of partial differential equations and time delay systems. The theoretical material is complemented by numerous examples illustrating how the semigroup theory provides a unified framework for the analysis of finite- and infinite-dimensional dynamical systems.

The topics include:

- Uniformly vs. strongly continuous semigroups of bounded linear operators on Banach spaces;
- Infinitesimal generators and resolvents;
- Generation theorems for C_0 -semigroups;
- Abstract Cauchy problems and well-posedness of evolution equations;
- Stability and exponential stability of semigroups;
- Spectral vs. Lyapunov criteria for stability;
- Operator Lyapunov equations and their relation to exponential stability;
- Some perturbation results;
- Evolution families for non-autonomous evolution equations;
- Periodic evolution equations and monodromy operators;
- Applications to the selected classes of PDEs, including heat, wave, and transport equations;
- Applications to time delay systems including construction of Lyapunov functionals, etc.

Prerequisites: basic knowledge of functional analysis and elementary ODE and PDE theory.

References

- [1] K.J. Engel & R. Nagel. *One-Parameter Semigroups for Linear Evolution Equations*. Graduate Texts in Mathematics, vol 194. Springer, New York, NY, 2000.
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- [3] J. Goldstein. *Semigroups of Linear operators & Applications*. Courier Dover Publications, 2017.
- [4] T. Eisner. *Stability of operators and operator semigroups*. Basel: Birkhäuser, 2010.
- [5] R.F. Curtain, & H. Zwart. *Introduction to Infinite-Dimensional Systems Theory: A State-Space Approach*. Vol. 71. Springer Nature, 2020.