Summer Semester 2025 S4B1 – Graduate Seminar on Analysis

Lyapunov methods for time delay systems

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Preliminary meeting: Tuesday 28th January at 14:15, Room 2.040, Endenicher Allee 60

If you cannot come to preliminary meeting but are interested in the seminar, please feel free to contact the organizers by the e-mail given above.

Time delay systems appear naturally in various applications such as engineering, robotics, epidemiology and, more generally, biology and population dynamics. Indeed, most processes in real life, whether it is the signal transmission in a feedback engineering system or many types of the chemical reactions, do not happen instantly, therefore delays must be taken into account in the modeling of such processes. Some examples of the models incorporating time delays can be found in the monographs [1, 2].

During this seminar, modern methods of the stability theory of time delay systems will be discussed with the focus on the Lyapunov methods following mainly the first two chapters of the monograph [3]. Since nonlinear systems can be studied by linearisation in many cases, a significant part of the seminar will be devoted to considering the class of linear systems. The class of homogeneous time delay systems, with a certain restriction on the homogeneity degrees of the right-hand sides, will be also briefly discussed.

For a class of linear time delay systems, the concept of the delay Lyapunov matrix plays a central role in our consideration. We will see that this matrix naturally appears during the construction of the Lyapunov functionals for time delay systems [4–6], and it serves as a time delay analogue of the solution to the classical Lyapunov matrix equation for delay free systems. We introduce and discuss two equivalent definitions of the delay Lyapunov matrix, and consider some numerical problems, such as robustness analysis and determination of the critical values of delays, where it can be useful. Finally, recent stability criteria for time delay systems expressed exclusively in terms of the delay Lyapunov matrix will be studied [7–9]. These criteria are mainly based on various discretization schemes for the Lyapunov functionals. Advantages and disadvantages of each particular discretization scheme with respect to a particular problem under consideration will be discussed in detail. It is worthy of mention that the stability criteria under discussion are verifiable in a finite number of mathematical operations, despite the fact that the class of systems with time delays represents a class of infinite-dimensional systems.

Prerequisites: basic knowledge of ODEs and linear algebra.

References

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[4] Y.M. Repin. Quadratic Lyapunov functionals for systems with delay. *Journal of Applied Mathematics and Mechanics*, 29, 669–672, 1966.

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[6] W. Huang. Generalization of Liapunov's theorem in a linear delay system. *J. of Mathematical Analysis and Applications*, 142, 83–94, 1989.

[7] S. Mondié, A.V. Egorov, & M. Gomez. Lyapunov stability tests for linear time-delay systems. *Annual Reviews in Control*, 54, 68–80, 2022.

[8] M. Bajodek, F. Gouaisbaut, & A. Seuret. Necessary and sufficient stability condition for time-delay systems arising from Legendre approximation. *IEEE Transactions on Automatic Control*, 68(10), 6262–6269, 2023.

[9] I.V. Alexandrova, & A.I. Belov. Synthesis of discretized Lyapunov functional method and the Lyapunov matrix approach for linear time delay systems. *Automatica*, 171, 111793, 2025.