Asymptotic Analysis. (V5B3).

Summer Semester 2026.

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Asymptotic methods are a set of techniques that allow to compute approximate formulas for the solutions of a large class of problems which contain some large or small parameter. The range of application of these methods includes areas like Classical Mechanics, Optics, Fluid Mechanics or Quantum Mechanics among others.

Asymptotic Analysis is one of the standard tools in Applied Mathematics, used to derive approximations for the solutions of many classes of equations. However, often the derived formulas are not mathematically rigorous and to obtain their precise mathematical meaning is a challenging problem. Frequently, the asymptotic formulas available for the study of one specific problem suggest interesting results in Analysis as well as interesting mathematical questions.

There is a relatively large diversity of Asymptotic Methods. A general strategy used to derive asymptotics is usually to rely on some explicit formulas which approximate the solution of the desired problem, at least in some regions of the space of variables. In many cases, one needs to connect different approximations of the solutions in different regions by means of a procedure called "matching". This concept plays an essential role in many asymptotic problems.

Some of the topics covered in this course are the following ones:

(a) Asymptotic series.
(b) Asymptotic approximations for solutions of linear and nonlinear ODEs.
(c) Multiple scales methods.
(d) Matching of solutions.
(e) WKB methods.
(f) Examples of asymptotic expansions in Partial Differential Equations.

Prerequisites. Basic knowledge of Analysis and Differential Equations.

References

[1] C. M. Bender and S. A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill, 1978.

[2] J. Kevorkian and J. D. Cole, Perturbation Methods in Applied Mathematics. Springer Verlag, 1980.

[3] M. H. Holmes, Introduction to Perturbation Methods, Springer Verlag, 2013.