Advanced Topics in Analysis and PDEs. (Summer Semester 2022).

Solitons and dispersive equations.

Juan J. L. Velázquez

This goal of the course is to describe the mathematical theory available for the study of some classes of dispersive equations. In the first part of the course the goal is to describe the method of the Inverse Scattering Transform (IST) and to study how this method can be used to solve the Cauchy problem for several Partial Differential Equations. In particular, applications of the IST to the solution of the KdV equation, the cubic nonlinear Schrödinger equation and other similar equations will be discussed. In this part of the course some topics like Lax pairs, Bäcklund transformations and their application to the study of completely integrable PDEs will be discussed.

In the second part of the course the general methods available to prove local and global well posedness of dispersive equations will be explained. In this part of the course topics like the Strichartz estimates and methods based in Fourier analysis will be studied.

Prerrequisites: Basic knowledge of the Theory of Partial Differential Equations and the Theory of Analytic Functions in $\mathbb{C}$.

References

[1] P. G. Drazin and R. S. Johnson, Solitons: an introduction. Cambridge Texts in Applied Mathematics, 1989.

[2] Terence Tao, Nonlinear Dispersive Equations: Local and Global Analysis. AMS, 2006.

[3] G. B. Whitham, Linear and nonlinear waves. New York, Wiley, 1974.