2022-2023, Winter semester
Seminar: Hydrodynamics limits of the Boltzmann equation

Introduction:

The Boltzmann equation plays a central role in mathematical physics, regarding very practical uses that can be done of it, as well as the mathematical phenomena this equation encodes. The Boltzmann equation describes dilute gases, that is, with a very low density. For instance, the upper layers of the atmosphere are well described by the equation. As for the mathematical interest, the Boltzmann equation describes irreversible behaviours, which is coherent with what we experience all in everyday life: the fluids have the natural trend to evolve towards rest states. The H-theorem is a rigorous statement of this phenomenon. However, the mathematical study of this equation is a very challenging topic, and many questions remain widely open.

On the other hand, it is also possible to study fluids, described for example by the Navier-Stokes equation, starting from the solutions of the Boltzmann equation, and considering that the fluids described are less and less dilute. Such an approach, called hydrodynamic limits, allows to justify rigorously the Navier-Stokes equation. This provides a partial solution to the Hilbert's sixth problem, which proposes to axiomatize Physics with Mathematics.

List of the topics:

- introduction to the Boltzmann equation (Basic properties, Global existence theory),
- the different models and equations in fluid mechanics,
- the details of the hydrodynamic limits.

References:

- The Boltzmann Equation and Its Hydrodynamic Limits, by François Golse, Chapter 3 in Handbook of Differential Equations: Evolutionary Equations, Volume 2, 2005.
- Hydrodynamic Limits of the Boltzmann Equation, by Laure Saint-Raymond, Lecture Notes in Mathematics n°1971, Springer-Verlag, 2009.

Prerequisites:

- Basics in Partial Differential Equations, and Functional Analysis.