Wave turbulence and kinetic theory

Graduate Seminar on Analysis (S4B1)

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Description

Wave Turbulence (WT) is a mathematical theory that utilizes methods coming from the kinetic theory of gases to the study of wave systems. The theory is very general and it can be applied to a large class of wave problems with small non-linearities. The resulting equations (which have similarities with the classical Boltzmann equation for gases) have interesting mathematical properties, but there are still relatively few rigorous mathematical results available.

The kinetic equations in the WT theory have many analogies with the Smoluchowski coagulation equation, which is used to describe aggregation phenomena in physical settings such as polymerization and blood coagulation.

During this seminar, we will explore intriguing phenomena that arise in these equations and the mathematical techniques employed to address these issues, which are closely related. Our focus will be to discover the connections between the WT kinetic equation and the classical coagulation equation.

Goals of the seminar

- Derivation of the kinetic equation for WT;
- Study of the mathematical properties of the WT equation;
- Connections and analogies between WT equation and the coagulation equations, including stationary solutions of these equations.

For a more detailed description of the course, please see the list of topics.

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Prerequisites

Basics of PDE and functional analysis.

Contact

For questions or if you would like to attend the seminar, please send an e-mail to cristian@iam.uni-bonn.de or to velazquez@iam.uni-bonn.de.

References

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