Models arising in geophysical fluid dynamics

Beyond the Euler and Navier-Stokes equation

Dr. Diego Alonso Orán, Dr. Amrita Ghosh
alonso@iam.uni-bonn.de, *ghosh@iam.uni-bonn.de*

Graduate Seminar on Analysis S4B1

Abstract

In this seminar we plan to study some problems of partial differential equations arising in geophysical fluid dynamics. More precisely, we will be concerned with a wide class of two dimensional transport equations that are called **active scalars** where the velocity is related to the transported quantity through an operator. Generally, this operator has a non-local nature which makes the analysis and understanding of the equations a big challenge. We will mostly focus on the following cases:

- **The surface quasi-geostrophic equation (SQG)**: describing the evolution of a surface buoyancy in a rapidly rotating, stratified potential vorticity fluid.
- **The incompressible porous media equation (IPM):** derived via Darcy's law for the evolution of a flow in a porous medium.
- **The Cordoba-Cordoba-Fontelos model (CCF):** motivated by Birkhoff-Rott equations modeling the evolution of vortex sheets with surface tension.

Beyond their own physical interest, the previous models serve as *toy models* for more complex three dimensional equations in fluid mechanics that are still far from being well understood. Therefore, the first part of the seminar will deal with the formal derivation of the previous models and their connection with well-known classical fluid equations as the Euler and Navier-Stokes equations [2, 3, 6]
In the second part of the seminar we will study several analytical properties of the models. In particular, we will study the well-posedness of the different systems (the existence and uniqueness of solutions) and the possible formation of singularities in finite time, [1, 4, 5].

Prerequisites: Basic knowledge of PDE's and Functional Analysis. There is no overlap with the *Seminar on Fluid Dynamics (S4B2)* which can rather serve as a supplementary course.
Prelminary meeting: The online preliminary meeting will be the 15 of February 2021 via Zoom.

References

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