The rigorous mathematical approach to Kinetic Theory of Gases and Plasmas.

V5B4 - Selected Topics in PDE and Mathematical Models

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Synopsis

Kinetic equations have been used for more than a century to describe the collective behaviour of many particle systems. At the end of the XIXth century J. C. Maxwell and L. E. Boltzmann addressed independently the problem of the mathematical description of classical dilute gases, in an attempt to produce a reduced kinetic picture emerging from the microscopic fundamental laws of classical mechanics.

The general idea underlying the study of kinetic equations, namely the methodology of the Kinetic Theory, is the following one. Many interesting systems in physics are constituted by a large number of identical components so that they are difficult to analyze from a mathematical point of view. At the same time we are not interested in a detailed description of the system but rather in its collective behavior. Therefore it is necessary to look for all the procedures leading to simplified models which preserve all the interesting physical informations of the original system, cutting away redundant information.

The two main examples we will use are the Vlasov equation and the Boltzmann equation. Specifically, the Vlasov equation is a time-reversible transport equation which determines the statistical properties of plasmas. This equation is widely used also in cosmology and astrophysics to study the formation and evolution of galaxies. It reads as

$$\partial_t f + v \cdot \nabla_x f + F(t,x) \cdot \nabla_v f = 0 \tag{1}$$
$$F(t,x) = -\nabla\phi *_x \rho, \quad \rho = \int f(t,x,v)dv$$

where $f : \mathbb{R}_+ \times \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}_+$, $f = f(t,x,v)$, is the probability density of finding a particle with position x and velocity v at time t, and ϕ is a smooth interaction potential.

The Boltzmann equation describes the evolution of a rarefied gas and reads as

$$\partial_t f + v \cdot \nabla_x f = Q(f,f) \tag{2}$$
$$Q(f,f)(t,x,v) = \int_{S^2} d\omega \int_{\mathbb{R}^3} dv_* B(v - v_*, \omega) \left\{ f(t,x,v')f(t,x,v'_*) - f(t,x,v)f(t,x,v_*) \right\}. \tag{3}$$

This course is intended to make an introductory review of the literature on kinetic equations. More precisely, we will deal with the well-posedness problem for the Vlasov equation (1) and the Boltzmann equation (2) by exploiting their structure and analytical properties which reflect some important physical features.

Moreover, we want to outline, with mathematical rigour, the limiting procedures which lead from the microscopic description based on the fundamental laws of mechanics (Newton equations) to a kinetic picture described by the nonlinear partial differential equations (1), (2). The exact derivation of these macroscopic evolution equations is one of the central problems in mathematical physics (Hilbert's sixth problem).

Prerequisites: basic knowledge of PDEs and functional analysis.

Time and place: Tuesdays 10 (c.t.) - 12, room 0.003 - Neubau

Literature sample:

1. C. Cercignani, R. Illner, M. Pulvirenti,
   The Mathematical Theory of Dilute Gases. Springer, Berlin, (1994)

2. Lecture notes