“Schrödinger operators with magnetic fields and applications to superconductivity”

Graduate seminar on Analysis (S4B2), Summer term 2020

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Abstract

The main topic of this seminar is the spectral problem for the magnetic Schrödinger hamiltonian of the form

$$\begin{cases} -(\nabla + iA)\cdot(\nabla + iA)\Psi = \lambda\Psi & \text{in } \Omega \subset \mathbb{R}^2 \\ \text{boundary condition on } \partial\Omega. \end{cases} \tag{1}$$

Here, the vector field $A : \Omega \to \mathbb{R}^2$ is such that $\nabla \times A = B$ in Ω, for a fixed *magnetic field* $B : \Omega \to \mathbb{R}^3$.

In particular, we are interested in the existence of eigenvalues $\lambda \in \mathbb{R}$ for which the corresponding solution Ψ in (1) is localized only in a specific portion of the domain Ω. For a given vector field B, both the geometry of the domain and the choice of boundary conditions give rise to different localization phenomena. For instance, in the case of Neumann boundary conditions and a strong magnetic field B that is perpendicular to the plane $\mathbb{R}^2$ in which the domain Ω lies, there exist eigenvalues whose eigenfunctions are mostly localized in the points of maximal curvature of the boundary $\partial\Omega$. In the same setting, but with Dirichlet boundary conditions, this is no longer true.

One of the main reasons for studying the localization phenomena of (1) comes from the theory of superconductivity described via the Ginzburg-Landau system. In the case of Neumann boundary conditions, indeed, the smallest eigenvalue of (1) and the localization regions of the corresponding eigenfunctions are strictly related to understanding the appearance of superconductivity at the critical regime [1]. In case of a sufficient number of talks and/or interested students, this part will also be covered in the seminar.

Prerequisites: Basic knowledge of functional analysis and PDEs.

Main references

[1] S. Fournais and B. Helffer. *Spectral Methods in Surface Superconductivity*, Progress in Nonlinear Differential Equations and Their Applications (2010), Vol. 77, Birkhuser Basel.

[1] For a more detailed, but still rather informal explanation, we refer to the introduction of [1].