"Interfaces and singularities in fluid dynamics"
Water waves and Muskat problem

Graduate seminar on Analysis (S4B2)

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Abstract

In this seminar we plan to study some problems of partial differential equations that arise in fluid mechanics. In particular, we are interested in models which involve the interaction between two incompressible fluids, or a fluid in vacuum. A big challenge in these settings is to describe the evolution of the interface between the different fluids considered or the fluid and vacuum.

We will mostly focus on [0.1, 0.2]:

(i) **Water waves:** The description of water waves in a dry region (vacuum), as described by a free-boundary problem for incompressible Euler;

(ii) **Muskat problem:** The interaction between two fluids of different density in a porous medium (e.g. water and oil in sand), described by means of Darcy's law.

One part of this seminar will be devoted to the derivation of the previous models: In the case of Darcy's law, for instance, this is obtained by means of homogenization for the Stokes equations in a perforated domain [1].

The second part of the seminar will focus on the study of the two types of singularities that the interfaces in (i)-(ii) may develop at a finite time $t > 0$: The *splash* and *splat* singularities. The splash-type singularity (Figure (a)) corresponds to the case where the fluid interface self-intersects at a single point. The splat-type singularity (Figure (b)), is a variation of the splash one in which the fluid interface self-intersects along an arc. The formation of these singularities will depend on the model and on the characteristics of the fluids considered [2,3,4].

(a) Splash singularity (b) Splat singularity

Prerequisites: Basic knowledge of PDEs and Functional Analysis.

Preliminary meeting on Wednesday 30th January at 2pm (c.t.), Room 2.040.

Main references

1) G. Allaire, *Continuity of the Darcy's law in the low-volume fraction limit*, Ann. della Scuola Normale Sup. di Pisa, Classe di Scienze 4^e série, tome 18, no. 4 (1991), pp. 475-499;

2) A. Castro, D. Córdoba, C. Fefferman, and F. Gancedo, *Splash singularities for the one-phase Muskat problem in stable regimes*, Arch. Ration. Mech. Anal., 222(1):213–243, 2016;

3) A. Castro, D. Córdoba, C. Fefferman, F. Gancedo, and J. Gómez-Serrano, *Finite time singularities for the free boundary incompressible Euler equations*, Ann. of Math. (2), 178(3):1061–1134, 2013;

4) D. Córdoba and T. Pernas-Castaño, *Non-splat singularity for the one-phase Muskat problem*, Trans. Amer. Math. Soc., 369(1):711–754, 2017.

For a first introductory reading

0.1) C. Fefferman, *Formation of Singularities in Fluid Interfaces*, J.È.D.P. (2012), Exposé no. II;

0.2) A. J. Majda and A. L. Bertozzi, *Vorticity and incompressible flow*, vol. 27 of Cambridge Texts in Applied Mathematics, Cambridge University Press, Cambridge, 2002.