

Lecture course V5B6 - Selected topics in Analysis and Calculus of Variations

Functions of Bounded Variation

Dr. Marco Bonacini

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Bonn University

Synopsis. The goal of the course is to give a general introduction to the theory of functions of bounded variation and of sets of finite perimeter. This theory provides the natural setup for several problems in the Calculus of Variations, in particular those characterized by the appearance of discontinuity surfaces. In this course I will present the main features of BV functions and the basic tools that the general theory makes available. The last part will be devoted to the discussion of a few applications to specific variational problems - in particular, the gradient theory of phase transition, and the Mumford-Shah functional (used in problems of image reconstruction and of fracture mechanics).

Tentative program:

- Preliminaries on Radon Measures.
- Definition and first properties of BV functions (compactness and lower semicontinuity, approximation, embedding theorems, trace, coarea formula)
- Sets of finite perimeter (the reduced boundary, De Giorgi structure theorem)
- Fine properties of BV functions (approximate continuity, jump set, decomposition of the gradient)
- Applications (gradient theory of phase transitions, the Mumford-Shah functional)

Prerequisites: basic courses in Measure Theory and Functional Analysis.

Suggested literature:

- L. AMBROSIO, N. FUSCO, D. PALLARA, *Functions of bounded variation and free discontinuity problems*. Oxford University Press, New York, 2000.
- A. BRAIDES, *Approximation of free discontinuity problems*. Springer-Verlag, Berlin, 1998.
- L.C. EVANS, R.F. GARIEPY, *Measure theory and fine properties of functions*. CRC Press, Boca Raton, FL, 1992.

Contact: bonacini@iam.uni-bonn.de

Time and place: Tuesdays, 10:15 – 12
Room 2.040