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Explicit methods in the theory of Partial Differential Equations

(lecture V5B1: advanced topics in Analysis and PDE)

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The purpose of this course is to describe several methods based in integral transformations which allow to obtain explicit representation formulas for a large class of differential and integral equations. In particular several problems which can be solved using the so-called Wiener-Hopf method will be studied in detail. These methods allow to solve explicitly many integral equations, boundary value problems with different types of boundary conditions and a wide class of integro-differential evolution equations. This approach is extensively used in order to obtain information for several problems of wave propagation, stability of solutions in kinetic equations, behaviour of solutions of elliptic and parabolic problems at boundary points where the type of boundary conditions change, probability theory and many others. Although most of the problems which can be studied with these methods are linear, there are some particular nonlinear problems which can be also studied with these techniques. Some of the topics covered in the course will be the following ones:

- Integral transforms (Laplace, Fourier and Mellin). Revision of complex variable.
- The Wiener-Hopf method.
- Application of the integral transform methods to problems of wave propagation and kinetic theory.
- Some examples of nonlinear problems with explicit solutions.

References:

Karl F. Graff, Wave motions in elastic solids, Oxford University Press,1975, Dover edition, 1991.

D. G. Crighton, A. P. Dowling, J. E. Ffowcs Williams, M.Heckl and F. G.Leppington, Modern Methods in Analytical Acoustics. Springer Verlag. 1992.

H. Hochstadt, Integral Equations,1973

B. Noble, Methods based on the Wiener-Hopf technique for the solution of partial differential equations. 1958.

V. E. Zakharov, Nonlinear waves and weak turbulence. AMS Translations, 1998.