

1) Solve the following difference equations:

$$\begin{aligned} a_{n+1} &= na_n = n^2 \\ a_{n+1} - \frac{na_n}{(n+1)} &= \frac{1}{n} \end{aligned}$$

2) The Bessel functions J_n satisfy the difference equation

$$J_{n+1}(z) - 2n \frac{J_n(z)}{z} + J_{n-1}(z) = 0$$

with $J_0(0) = 1$ and $J_n(0) = 0$ if $n \neq 0$. Define the generating function $f(x, z) = \sum_{n=-\infty}^{\infty} x^n J_n(z)$.

- Show that $f(x, z) = \exp\left(\frac{z}{2}\left(x - \frac{1}{x}\right)\right)$.
- Show that $J_n(z) = J_n(-z) = (-1)^n J_n(z)$.
- Show that $1 = J_0(z) + 2 \sum_{n=1}^{\infty} J_{2n}(z)$, $z = 2 \sum_{n=0}^{\infty} (2n+1) J_{2n+1}(z)$.