Workshop on

"The Mathematical Theory of Particle Suspensions"

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organized by
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ABSTRACTS

Kleber Carrapatoso (Montpellier)

Mean-field limit of the Stokes equation around random spheres

Abstract: This talk is concerned with the Stokes equation in the whole space around N spherical obstacles. I will show that in the mean-field limit and choosing obstacles randomly, one can derive the Stokes-Brinkman equation. This is a joint work with Matthieu Hillairet.

Laurent Desvillettes (Paris Diderot)

About thick sprays

Abstract: When a dispersed phase (droplets or dust specks) is coupled to a surrounding gas by the volume fraction that it occupies as well as by the drag force, the flow is sometimes called a thick spray. It can be modeled at the mesoscopic level by a system in which the volume fraction is one of the unknown. This system has not been studied a lot at the mathematical level. We propose here to review some of the results which are known, and to explain the formal links of the mesoscopic modeling with other levels of modeling.

David Gérard-Varet (Paris Diderot)

The effective viscosity of suspensions beyond Einstein's formula

Abstract: We will discuss the effective viscosity generated by a suspension of solid particles in a Stokes flow. We will focus on the regime of small solid volume fraction ϕ. In this setting, a linear approximation of the effective viscosity is provided by Einstein's formula: $\mu_{eff} = \mu + \frac{5}{2}\phi\mu$. We will discuss in this talk the quadratic correction, for which pair interactions must be taken into account.

We will show how the mathematical analysis developped by S. Serfaty and co-authors on Coulomb gases can be applied, providing explicit formulas. This is a joint work with M. Hillairet.

Antoine Gloria (Paris Sorbonne)

A homogenization approach to effective properties in dilute media

Abstract: Consider a background medium perturbed by inclusions of a different medium. Can this heterogeneous medium be replaced by some effective medium at large scales? If yes, in the regime when inclusions are scarce, can one get more information in form of an explicit approximation of this effective medium in terms of the concentration of inclusions (a perturbative result)? In the physics literature, the existence of an effective medium is implicitly taken for granted, and the focus is on explicit approximations in the dilute regime. The first such result is the celebrated Clausius-Mossotti-Rayleigh-Maxwell formula derived in the XIXth century for the effective conductivity (and permeability) of an isotropic medium perturbed by spherical inclusions of a different isotropic medium. In the first half of the XXth century, Einstein derived a similar formula for the effective viscosity of a Stokes fluid with colloidal spherical suspensions. In the second half of the XXth century, Batchelor obtained a formula of a different flavour for the effective velocity of spherical suspensions sedimenting in a Stokes fluid. For these three physical models, I will give a precise answer to the first question, i.e. the existence of effective properties. To this aim I will make a detour through (quantitative) stochastic homogenization, and revisit the puzzling Caflisch-Luke paradox in sedimentation. I will then turn to the second question, i.e. the approximation of effective properties in the dilute regime. For the latter, geometric properties of the distribution of inclusions come into play in a subtle way. I will discuss in particular the analyticity of the effective properties with respect to suitable dilution parameters.

This talk is based on joint works with Mitia Duerinckx (CNRS, Orsay) and Jules Pertinand (Sorbonne Université).

François Golse (Paris, Ecole polytechnique)

From the kinetic theory of gases to models for aerosol flows

Abstract: The Vlasov-Stokes or the Vlasov-Navier-Stokes systems are used in the description of thin sprays or aerosol flows. This talk explains how these systems can be derived from the system of Boltzmann equations for binary gas mixtures. One component of this mixture is the dispersed phase (particles or droplets in the aerosol), while the other component is the propellant gas. The coupling between these two species is only due to the friction of the propellant gas on the dispersed phase. The Vlasov-Stokes or Vlasov-Navier-Stokes systems are obtained in appropriate scaling regimes from the Boltzmann system by a rather delicate asymptotic analysis.

Christiane Helzel (Düsseldorf)

Modelling and simulation of sedimentation in suspensions of rod-like particles

Abstract: We consider a kinetic multi-scale model which describes the sedimentation process in suspensions of rod-like particles. The full model is a time dependent five-dimensional coupled system of partial differential equations, which will be derived and analysed in Tzavaras' talk. Due to the high dimensionality of the problem, detailed numerical simulations are cumbersome. Therefore, we are interested in reduced models, which can be derived by considering moment equations. Our goal is the derivation of a method which adapts the model locally based on accuracy requirements. As a first step in this direction, we consider a simplified situation and compare discretizations of the full model with discretizations of different moment systems.

Matthieu Hillairet (Montpellier)

Homogenization of the Euler equations in perforated domains

Abstract: In this talk, we compare two models for the motion of an inviscid fluid in a perforated domain. In the first one, we consider the Euler equations set on $\mathbb{R}^2$ deprived from a finite number of obstacles at rest. In the second one, the porous medium is taken into account via its volume fraction. Relying on the analogies with the effective viscosity problem, we give quantitative bounds on the difference between solutions to the two models. This result has been obtained in collaboration with C. Lacave and Di Wu.

Richard Höfer (Bonn)

The method of reflections and its applications for inertialess suspensions

Abstract: We consider suspensions of inertialess rigid sedimenting particles in Stokes flows. We show how the method of reflections can be used to rigorously derive a coupled transport-Stokes system for the dynamics of the system in the limit of many small particles, in the regime where the volume fraction ϕ of the particles tends to zero.

The method of reflections is a method to expand the solution operator to a linear PDE in domains perforated by many disjoint sets in terms of the solution operator when only one of these sets is present. We discuss the convergence properties of the method and how it could be used to tackle related problems.

Pierre-Emmanuel Jabin (Maryland)

Quantifying complexity and propagation of chaos in large stochastic systems of interacting particles

Abstract: I will present some recent results, obtained with Z. Wang and D. Bresch and Z. Wang, on large stochastic many-particle or multi-agent systems. Because such systems are conceptually simple but exhibit a wide range of emerging macroscopic behaviors, they are now employed in a wide range of applications from Physics (plasmas, galaxy formation...) to the Biosciences, Economy, Social Sciences...

However the number of agents or particles is typically quite large, with 10^{20}-10^{25} particles in many Physics settings for example. Analytical or numerical studies of such systems are potentially very complex and one key question is whether it is possible to reduce this complexity, notably with the notion of propagation of chaos (almost propagation of independence). Classical trajectorial approaches have been able to provide quantitative answers to propagation of chaos when the interactions' kernel between particles are smooth. Unfortunately many realistic settings instead involve singular interactions for which we did not have any quantitative estimates in the stochastic case.

We have introduced a novel analytical method, based on explicit relative entropy bounds. This method required the development of new and delicate large deviation inequalities but led to the derivation of the mean-field limit for attractive singular interactions such as the ones present in the Keller-Segel model for chemotaxis and some Coulomb gases

Marta Leocata (Lyon)

The Vlasov-Fokker-Planck-Navier-Stokes system as a scaling limit of particles in a fluid

Abstract: The PDEs system Vlasov-Fokker-Planck-Navier-Stokes (VFPNS) is a model describing particles in a fluid, where the interaction particles- fluid is described by a drag force called Stokes drag force. In the talk I will present a particle system interacting with a fluid that converges in a suitable probabilistic sense to the (VFPNS) system. The interaction between particles and fluid is described by Stokes drag force. I will show how the empirical measure S^N of particles converges to the Vlasov-Fokker-Planck component of the system and the velocity of the fluid u^N coupled with the particles converges in the uniform topology to the Navier-Stokes component.

Amina Mecherbet (Paris Diderot)

A model for suspension of clusters of particle pairs

Abstract: We consider N clusters of pairs of particles sedimenting in a viscous fluid. The particles are assumed to be rigid spheres and inertia of both particles and fluid are neglected. The distance between each two particles forming the cluster is comparable to their radii while the minimal distance between the pairs is as least of order $N^{-1/2}$. We assume that the dilution regime is conserved in finite time and show that, at the mesoscopic level, the dynamics are modeled using a transport-Stokes equation describing the time evolution of the position and orientation of the clusters. Under the additional assumption that the minimal distance is at least of order $N^{-1/3}$, we investigate the case where the orientation of the cluster is initially correlated to its position. In this case, a local existence and uniqueness result for the limit model is provided.

Ayman Moussa (Paris Sorbonne)

Some recent results and open questions about the Vlasov-Navier-Stokes system

Abstract: The talk will focus on a fluid-kinetic model known as the Vlasov-Navier-Stokes system. I will present recent results established in collaboration with Daniel Han-Kwan and Ivan Moyano about the long time behavior of weak solutions and explain the very partial results known concerning the derivation of this system and some variants. This include also a joint work with Matthieu Hillairet and Franck Sueur.

Valeria Ricci (Palermo)

Derivation of macroscopic models for thin sprays from multiphase Boltzmann models with inelastic collision kernel

Abstract: We shall illustrate the formal validation of macroscopic models for thin sprays, consisting of a Vlasov-type equation (describing the dispersed phase) coupled to a hydrodynamic equation (describing the surrounding gas), from multiphase Boltzmann models with inelastic collision kernel in the Stokes, Navier-Stokes and Euler asymptotics for the multiphase Boltzmann model.

Richard Schubert (Aachen)

A local version of Einstein's formula

Abstract: In 1906 Einstein gave a, now famous, formula for the effective viscosity of dilute suspensions, that is $\mu' = m(1 + \frac{5}{2}\phi + o(\phi))$, where ϕ is the volume fraction of the suspended particles. In this talk I will present a result on the local validity of the formula, i.e. on the level of the Stokes equation (with variable viscosity). In particular I will discuss the dipole approximation of the flow field of the suspension and the method of reflections in this setting. Finally I will comment on the interplay between Einstein's formula and sedimentation. This talk is based on joint work with Richard Höfer and Barbara Niethammer.

Athanasios Tzavaras (KAUST)

Kinetic models for the description of sedimenting suspensions

Abstract: I review some works on modeling and the mathematical theory for dilute suspensions of rigid rods. Such problems appear in modeling sedimentation of suspensions of particles. Similar in spirit models are also used for modeling swimming micro-organisms. Here, we focus on a class of models introduced by Doi and describing suspensions of rod–like molecules in a solvent fluid. They couple a microscopic Fokker-Planck type equation for the probability distribution of rod orientations to a macroscopic Stokes flow. One objective is to compare such models with traditional models used in macroscopic viscoelasticity as the well known Oldroyd model. In particular: For the problem of sedimenting rods under the influence of gravity we discuss the instability of the quiescent flow and the derivation of effective equations describing the collective response. We derive two such effective theories: (i) One amounts to a classical diffusive limit and produces a Keller-Segel type of model. (ii) A second approach involves the derivation of a moment closure theory and the approximation

of moments via a quasi-dynamic approximation. This produces a model that belongs to the class of flux-limited Keller-Segel systems. The two theories are compared numerically with the kinetic equation. (joint work with Christiane Helzel, Univ. Duesseldorf).