Summer Semester 2025 S4B1 – Graduate Seminar on Analysis

Lyapunov methods for time delay systems

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List of the topics:

Topic 0 (my). Introduction to the stability theory of time delay systems. Examples of Lyapunov functions for ODEs.

Topic 1 (optional). Lyapunov stability theorems for a general class of nonlinear time delay systems (Sections 1.4 and 1.5 in [3]).

Topic 2. A class of linear time delay systems. Fundamental matrix. Cauchy Formula. Origins of the theory of functionals with prescribed derivatives: How does the Cauchy Formula give rise to an explicit expression for the Lyapunov functional? (Sections 2.1–2.5 in [3]).

Topic 3. The concept of Delay Lyapunov Matrix. Two definitions and their equivalence for a class of exponentially stable systems (Sections 2.6–2.8 in [3]).

Topic 4 (optional). Existence, uniqueness and computational issue for the Delay Lyapunov Matrix (Sections 2.9–2.10 in [3]).

Topic 5 (optional). Applications of the Delay Lyapunov Matrix. Construction of the exponential estimates for solutions. Robust stability analysis. Computation of the critical values of delay (Sections 2.11–2.12 in [3]).

Topic 6 (optional). Necessary and sufficient stability conditions expressed exclusively in terms of the Delay Lyapunov Matrix [7–9].

Topic 7* (optional). Some engineering and biological examples of models involving systems with time delays ([1,2] and other sources).

References

[1] S.-I. Niculescu. *Delay effects on stability: A robust control approach.* Springer Science & Business Media, vol. 269, 2001.

[2] E. Fridman. *Introduction to time-delay systems: Analysis and control.* Birkhäuser, Basel, 2014.

[3] V.L. Kharitonov. *Time-delay systems: Lyapunov functionals and matrices.* Birkhäuser, Basel, 2013.

[4] Y.M. Repin. Quadratic Lyapunov functionals for systems with delay. *Journal of Applied Mathematics and Mechanics*, 29, 669–672, 1966.

[5] E.F. Infante, & W.B. Castelan. A Liapunov functional for a matrix difference-differential equation. *J. Differential Equations*, 29(3), 439–451, 1978.

[6] W. Huang. Generalization of Liapunov's theorem in a linear delay system. *J. of Mathematical Analysis and Applications*, 142, 83–94, 1989.

[7] S. Mondié, A.V. Egorov, & M. Gomez. Lyapunov stability tests for linear time-delay systems. *Annual Reviews in Control*, 54, 68–80, 2022.

[8] M. Bajodek, F. Gouaisbaut, & A. Seuret. Necessary and sufficient stability condition for time-delay systems arising from Legendre approximation. *IEEE Transactions on Automatic Control*, 68(10), 6262–6269, 2023.

[9] I.V. Alexandrova, & A.I. Belov. Synthesis of discretized Lyapunov functional method and the Lyapunov matrix approach for linear time delay systems. *Automatica*, 171, 111793, 2025.