Abstracts

Decoupling and decay of two-point functions in a two-species TASEP

Patrik L. Ferrari
(joint work with Sabrina Gernholt)

We consider a special case of an n-component lattice gas models on $\mathbb{Z}$. These consist of particles of types $\alpha \in \{1,\dots,n\}$ with at most one particle per site evolving with nearest-neighbor jumps. Let $\eta_\alpha(j,t)=1$ if an α-particle is at site j at time t and $\eta_\alpha(j,t)=0$ otherwise. We consider the case where the jumps rates are local and translation-invariant. Assume that for any fixed $\rho=(\rho_1,\dots,\rho_n)$ with $\rho_i\in[0,1]$ and $\sum_{i=1}^n \rho_i \leq 1$, there exists an ergodic, translation-invariant stationary measure μ_ρ with $\mathbb{E}_{\mu_\rho}(\eta_\alpha)(j,t)=\rho_\alpha$.

The main observable we study is the two-point function $S=(S_{\alpha,\beta})_{1\leq\alpha,\beta\leq n}$, with

$$S_{\alpha,\beta}(j,t)=\mathbb{E}_{\mu_\rho}(\eta_\alpha(j,t)\eta_\beta(0,0))-\rho_\alpha\rho_\beta. \tag{1}$$

The *nonlinear fluctuating hydrodynamic theory* (NLFH) gives a prediction on the large time behavior of the two-point function (see [7, 9, 11, 12, 13, 14] for related papers). Denote by $\mathrm{j}_\alpha(\rho)$ be the expected infinitesimal current of α-particles under μ_ρ. Let $C=C^T=\sum_j S(j,t)=\sum_j S(j,0)$ be the susceptibility matrix and define the matrix A with components $A_{\alpha,\beta}(\rho)=\frac{\partial}{\partial\rho_\beta}\mathrm{j}_\alpha(\rho)$. It is known that $AC=CA^T$ and assume $C>0$ to avoid having components that do not evolve over time.

In order to see something meaningful one needs to consider appropriate linear combinations of the types of particles, the so-called *normal modes.* More precisely, one needs to find a matrix R such that

$$RAR^{-1}=\mathrm{diag}(v_1,\dots,v_n) \quad \text{and} \quad RCR^T=\mathbf{1}. \tag{2}$$

Then the normal modes are given by $\xi=R\eta$ and v_α is the speed of propagation of the mode ξ_α. The two-point function of the normal modes is given by

$$S^{\#}_{\alpha,\beta}(j,t)=(RSR^T)_{\alpha,\beta}(j,t)=\mathbb{E}_{\mu_\rho}(\xi_\alpha(j,t)\xi_\beta(0,0))-(R\rho)_\alpha(R\rho)_\beta. \tag{3}$$

The prediction (as stated in [7]) is the following: assume that $v_1,\dots,v_n$ are all distinct, then there exists some (explicit) constants $\lambda_1,\dots,\lambda_n$ such that

$$(\lambda_\alpha t)^{2/3}S^{\#}_{\beta,\gamma}(v_\alpha t+w(\lambda_\alpha t)^{2/3},t)\simeq\delta_{\beta,\alpha}\delta_{\gamma,\alpha}f_{\mathrm{KPZ}}(w) \tag{4}$$

as $t\to\infty$. Here, the scaling function f_{KPZ} is the one of one-dimensional system [10] and it is given by

$$f_{\mathrm{KPZ}}(w)=\frac{1}{4}\frac{d^2}{dw^2}\int_{\mathbb{R}} s^2 dF_{\mathrm{BR},w}(s), \tag{5}$$

where $F_{\mathrm{BR},w}$ is the Baik-Rains distribution with parameter w [4].

In our work [5] we consider a model with $n = 2$: a two-species totally asymmetric simple exclusion process with first class particles η_1 and second class particles η_2. The normal modes are given by

$$\xi_1(j,t) = \frac{\eta_1(j,t)}{\sqrt{\rho_1(1-\rho_1)}}, \quad \xi_2(j,t) = \frac{\eta_1(j,t)+\eta_2(j,t)}{\sqrt{(\rho_1+\rho_2)(1-\rho_1-\rho_2)}}, \tag{6}$$

and the speeds are $v_1 = 1 - 2\rho_1$, $v_2 = 1 - 2(\rho_1 + \rho_2)$.

Our main result is the following.

Theorem 1. *Given a speed v, define*

$$\mathcal{S}_v^{\#}(\phi) = \lim_{t\to\infty} t^{-2/3} \sum_{w \in t^{-2/3}\mathbb{Z}} \phi(w) t^{2/3} S^{\#}(vt + wt^{2/3}, t) \tag{7}$$

for ϕ smooth test functions with compact support. Then we have the following cases:

(a) if $v \notin \{v_1, v_2\}$,

$$\mathcal{S}_v^{\#}(\phi) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

(b) if $v = v_1$, then

$$\mathcal{S}_v^{\#}(\phi) = \begin{pmatrix} \chi_1 \int_{\mathbb{R}} \phi(w) \lambda_1^{-2/3} f_{\mathrm{KPZ}}(\lambda_1^{-2/3} w) dw & 0 \\ 0 & 0 \end{pmatrix}$$

with $\chi_1 = \rho_1(1-\rho_1)$ and $\lambda_1 = 2\sqrt{2\chi_1}$,

(c) if $v = v_2$, then

$$\mathcal{S}_v^{\#}(\phi) = \begin{pmatrix} 0 & 0 \\ 0 & \chi_2 \int_{\mathbb{R}} \phi(w) \lambda_2^{-2/3} f_{\mathrm{KPZ}}(\lambda_2^{-2/3} w) dw \end{pmatrix}$$

with $\chi_2 = (\rho_1+\rho_2)(1-\rho_1-\rho_2)$ and $\lambda_2 = 2\sqrt{2\chi_2}$.

The weak convergence of the diagonal terms was shown in [3] building on [8, 10]. In our paper [5] we prove that the off-diagonal terms vanishes in the large time limit. A similar result for ASEP under double scaling limit has been obtained in [1].

In order to prove our result, we derived a new identity [5, Proposition 1.2]

$$S_{1,2}^{\#}(\tilde{x}+i, \tilde{t}) + S_{2,1}^{\#}(x+i, t) = \frac{1}{4} \Delta \mathrm{Cov}\left(h^1(\tilde{x}+i, \tilde{t}), h^{1+2}(x+i, t)\right), \tag{8}$$

where h^1 (resp. h^{1+2}) is the standard height function for first (resp. first plus second) class particles and Δ is the discrete Laplacian. Then the main steps are the following:

(a) Suppose that $\mathrm{Supp}(\phi) \subset [-L, L]$ and take $x = v_2 t$, $i = wt^{2/3}$, $\tilde{x} = 2Lt^{2/3}$ and $\tilde{t} = 0$. Using the properties of the stationary measure of the multi-species TASEP, we first get an a-priori bound on $S_{1,2}^{\#}(\tilde{x}+i, \tilde{t})$, so that we need to consider only the second term.

(b) We perform summation by parts to move the discrete Laplacian to the test function ϕ: it remains to control $\mathrm{Cov}\left(h^1(\tilde{x}+i, \tilde{t}), h^{1+2}(x+i, t)\right)$.

(c) Using the queuing representation of the stationary measure [2, 6], we see that $h^1((2L+w)t^{2/3},0) = \tilde{h}^1((2L+w)t^{2/3},0) + R((2L+w)t^{2/3})$ where $\tilde{h}^1$ is independent of h^{1+2} (thus the covariance is zero) and the remainder term $|R| \ll t^{1/3}$. By Cauchy-Schwarz one finally control the covariance between R and h^{1+2}.

References

[1] A. Aggarwal, I. Corwin, and M. Hegde, *Scaling limit of the colored ASEP and stochastic six-vertex models*, arXiv:2403.01341v2 (2024).

[2] O. Angel, *The stationary measure of a 2-type totally asymmetric exclusion process*, J. Comb. Theory Ser. A **113** (2006), 625–635.

[3] J. Baik, P.L. Ferrari, and S. Péché, *Convergence of the two-point function of the stationary TASEP*, Singular Phenomena and Scaling in Mathematical Models, Springer, 2014, pp. 91–110.

[4] J. Baik and E.M. Rains, *Limiting distributions for a polynuclear growth model with external sources*, J. Stat. Phys. **100** (2000), 523–542.

[5] P.L. Ferrari and S. Gerholt, *Decoupling and decay of two-point functions in a two-species TASEP*, preprint, arXiv:2504.00765 (2025).

[6] P.A. Ferrari and J.B. Martin, *Stationary distribution of multi-type totally asymmetric exclusion processes*, Ann. Probab. **35** (2007), 807–832.

[7] P.L. Ferrari, T. Sasamoto, and H. Spohn, *Coupled Kardar-Parisi-Zhang equations in one dimension*, J. Stat. Phys. **153** (2013), 377–399.

[8] P.L. Ferrari and H. Spohn, *Scaling limit for the space-time covariance of the stationary totally asymmetric simple exclusion process*, Comm. Math. Phys. **265** (2006), 1–44.

[9] C.B. Mendl and H. Spohn, *Dynamic correlators of Fermi-Pasta-Ulam chains and nonlinear fluctuating hydrodynamics*, Phys. Rev. Lett. **111** (2013), 230601.

[10] M. Prähofer and H. Spohn, *Exact scaling function for one-dimensional stationary KPZ growth*, J. Stat. Phys. **115** (2004), 255–279.

[11] H. Spohn, *Nonlinear fluctuating hydrodynamics for anharmonic chains*, J. Stat. Phys. **154** (2014), 1191–1227.

[12] H. Spohn, *Fluctuating hydrodynamics approach to equilibrium time correlations for anharmonic chains*, Thermal transport in low dimensions: from statistical physics to nanoscale heat transfer, in Lect. Notes Phys. (2016), 107–158.

[13] H. Spohn and G. Stoltz, *Nonlinear fluctuating hydrodynamics in one dimension: the case of two conserved fields*, J. Stat. Phys. **160** (2015), 861–884.

[14] H. van Beijeren, *Exact results for anomalous transport in one-dimensional hamiltonian systems*, Phys. Rev. Lett. **108** (2012), 180601.

Reporter: