Abstracts

Introduction to the Kardar-Parisi-Zhang universality class

Patrik L. Ferrari

In the introductory lecture we describe the Kardar-Parisi-Zhang (KPZ) universality class of stochastic growth models, some of the large time limit processes, connections with interacting particle systems and directed polymers in a random environment. Finally we describes some useful techniques.

The KPZ universality class contains models describing the evolution of an interface given by a height function $x \mapsto h(x,t)$. Here we focus on space x being one-dimensional. The prototypical effective equation is the KPZ equation

$$\partial_t h = \frac{1}{2}\partial_x^2 h + \frac{1}{2}(\partial_x h)^2 + \xi$$

where ξ is the white noise. As written, the KPZ equation ill-defined since a stationary solution is a two-sided Brownian motion (with diffusion constant 2) and thus the term $(\partial_x h)^2$ is ill-defined. A way to make sense of it is to regularize the noise in space, $\xi \to \xi_\epsilon$, and take $\epsilon \to 0$, but we do not enter in these details in the lecture.

One can also consider discrete (in space and/or time) models, which have the *same physical properties* as the KPZ equations. Then by universality one expects the same large time limit process of the interface.

The connection with the directed polymers goes by considering $Z = \exp(h)$ and first solve the stochastic heat equation with multiplicative noise,

$$\partial_t Z = \frac{1}{2}\partial_x^2 Z + Z\xi$$

with initial condition $Z(x,0) = \exp(h(x,0))$ and then set $h(x,t) = \ln Z(x,t)$. This is called the Cole-Hopf solution of the KPZ equation. Due to a Feynman-Kac representation, we have an interpretation of Z a the partition function of a Brownian motion (the directed polymer) in a random environment (the white noise). The discrete analogue at zero temperatures is the so-called last passage percolation model (LPP).

The other connection is to interacting particle systems, more specifically with exclusion processes (e.g., the totally asymmetric simple exclusion process (TASEP) or the partially asymmetric version of it (ASEP)). These models can be seen as random interfaces by setting the discrete gradient to be ± 1 depending whether there is a particle or not, and the height function increment during time t is given by twice the current of particles.

Some of the limit processes and properties are discussed, namely for step initial condition $h(x,0) = |x|$ or flat initial condition $h(x,0) = 0$. These are called the Airy_2 and Airy_1 processes. For general simple exclusion processes there is a clear KPZ scaling theory which predict how to compute the non-universal scaling coefficients.

Two methods which have been quite useful to study large time asymptotics of observables are also discussed. One is the slow decorrelation, which tells us that along characteristic lines (of the macroscopic PDE) the fluctuations decorrelate only over macroscopic times, unlike the spatial correlations which are of order $t^{2/3}$. A second technique is the comparison with stationarity, first used in LPP models and then for TASEP height function. This allows to control the local increment of the height function for generic initial conditions with the increments of stationary initial conditions (which are explicit). In particular, to show tightness of the limit processes.

Reporter: