Abstracts

Time-time covariance for last passage percolation with generic initial profile

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(joint work with Alessandra Occelli and Herbert Spohn)

This talk is based on the papers [10] with H. Spohn and [9] with A. Occelli. In [10] we made some well-funded predictions on the time-time correlations for models in the Kardar-Parisi-Zhang universality class, while in [9] we provide mathematical proofs of the results presented below.

Stochastic growth models in the Kardar-Parisi-Zhang (KPZ) universality class [14] on a one-dimensional substrate are described by a height function $h(x,t)$ with x denoting space and t time. The height function evolves microscopically according to a random and local dynamics, while on a macroscopic scale the evolution is a deterministic PDE and the limit shape is non-random. In particular, if the speed of growth as a function of the gradient of the interface is a strictly convex or concave function, then the model is in the KPZ universality class. One expects large time universality under an appropriate scaling limit.

Away from shocks, the fluctuations of the height functions growth as $t^{1/3}$, while the spatial correlations are of order $t^{2/3}$. Furthermore, along space-time trajectories given by the characteristic lines of the PDE for the macroscopic evolution, non-trivial correlations survive on the macroscopic time scale, i.e., on scales of order t [8, 5].

The study of the time-time process is recent. On the experimental and numerical simulation side observables like the persistence probability or the covariance of an appropriately rescaled height function have been studied [19, 17, 18, 16]. On the analytic and rigorous side, the two-time joint distribution of the height function is known for special initial conditions: Johansson, later with Rahman, analyzed a model on full space [11, 12, 13], while Baik and Liu considered a model on a torus [2, 1]. For general (random) initial conditions exact formulas on the joint distributions are not yet available. Also, the analysis of the covariance starting from the available formulas [13, 12, 1] seems to be a very difficult task.

In [10] and [9] we consider the last passage percolation as model with generic initial conditions. The predictions of [10] are made under the assumptions that the exchange of the large time limit and maximum over sums of Airy processes and that the covariances of the rescaled processes converges holds. In [9] we provide mathematical proofs, using (1) the method of Corwin, Liu and Wang [7], who lifted the finite-dimensional slow-decorrelation result of [8, 5] to a functional slow-decorrelation statement, and (2) the method of comparison with stationarity developed by Cator and Pimentel [4, 15].

The model is the following. Consider a collection of i.i.d. random variables $\omega_{i,j}, i, j \in \mathbb{Z}$ with exponential distribution of parameter one. An *up-right path* $\pi = (\pi(0), \pi(1), \ldots, \pi(n))$ on $\mathbb{Z}^2$ from a point A to a point E is a sequence of points in $\mathbb{Z}^2$ with $\pi(k+1)-\pi(k) \in \{(0,1),(1,0)\}$, with $\pi(0)=A$ and $\pi(n)=E$, and n is called the length $\ell(\pi)$ of π. Given a set of points S_A with some random variables (not necessarily independent) h^0 on S_A, but independent of the ω's, and given a point E, one defines the last passage time $L_{S_A \to E}$ as

$$L_{S_A\to E} = \max_{\substack{\pi:A\to E\\ A\in S_A}} \left(h^0(\pi(0)) + \sum_{1\le k\le n} \omega_{\pi(k)} \right). \tag{1}$$

Here we consider $S_A = \mathcal{L} := \{(i,j) \in \mathbb{Z}^2 \,|\, i+j=0\}$ for the point-to-point geometry and $S_A = \{(0,0)\}$, $E = (\tau N, \tau N)$ for the remaining geometries. Define the limiting rescaled LPP time

$$\tau \mapsto \chi^\star(\tau) = \lim_{N\to\infty} \frac{L^\star_{S_A\to E} - 4\tau N}{2^{4/3} N^{1/3}}, \tag{2}$$

where the superscript $\star$ denotes the different configurations, point-to-point ($\bullet$), point-to-line ($\diagdown$), stationary ($\mathcal{B}$) and random (σ), which are given as follows:

- *Point-to-point:* $S_A = \{(0,0)\}$ and $h^0 = 0$,
- *Point-to-line:* $S_A = \mathcal{L}$ and $h^0 = 0$.
- *Stationary:* $S_A = \mathcal{L}$ and h^0 as follows. Let $\{X_k, Y_k\}_{k\in\mathbb{Z}}$ be i.i.d. random variable Exp(1/2)-distributed. Then define

$$h^0(x,-x) = \begin{cases} \sum_{k=1}^{x} (X_k - Y_k), & \text{for } x \ge 1, \\ 0, & \text{for } x = 0, \\ -\sum_{k=x+1}^{0} (X_k - Y_k), & \text{for } x \le -1. \end{cases} \tag{3}$$

- *A family of random initial conditions.* We consider the case where for a given $\sigma \ge 0$, h^0 is given by (3) multiplied by σ. Clearly, the cases $\sigma = 0$ and $\sigma = 1$ correspond to the flat and to the stationary cases.

The main results proven in [9] are the following (in [9] the results are extended to points in neighborhoods of the characteristics as well):

Theorem 1. *For the stationary LPP, the covariance of the limiting height function for all $\tau \in (0,1)$ can be expressed as*

$$\mathrm{Cov}\left(\chi^{\mathcal{B}}(\tau), \chi^{\mathcal{B}}(1)\right) = \frac{1+\tau^{2/3} - (1-\tau)^{2/3}}{2} \mathrm{Var}(\xi_{\mathrm{BR}}), \tag{4}$$

where ξ_{BR} is a Baik-Rains distributed random variable.

Theorem 2. *As $\tau \to 1$ we have, for $\star = \{\bullet, \diagdown, \mathcal{B}\}$,*

$$\mathrm{Cov}\left(\chi^\star(\tau), \chi^\star(1)\right) = \frac{1+\tau^{2/3}}{2} \mathrm{Var}\left(\chi^\star(1)\right) - \frac{(1-\tau)^{2/3}}{2} \mathrm{Var}\left(\xi_{\mathrm{BR}}\right) + \mathcal{O}(1-\tau)^{1^-}. \tag{5}$$

Here $\chi^\bullet(1)$ (resp. $2^{2/3}\chi^{\diagdown}(1)$) is distributed according to a GUE (resp. GOE) Tracy-Widom law.

Shortly after finishing our paper, for the point-to-point geometry, Basu and Ganguly obtained the same exponents for the behaviour at close or far away points [3]. Unlike in our paper, they did not identify the prefactor, but on the other hand, their result are non-asymptotic as well. Very recently, a result analogue to [3] for the KPZ equation with sharp wedge initial condition has been obtained in [6].

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