Abstracts

Around the universality of the Airy_1 process

PATRIK L. FERRARI
(joint work with Alexei Borodin, Tomohiro Sasamoto)

Half a decade ago, Prähofer and Spohn discovered the Airy_2 process in a surface growth model (the polynuclear growth (PNG) model) [12]. It is a universal process. It appears in directed last passage percolation percolation, various discrete growth models, domino tiling, random matrix theory (GUE Dyson's Brownian Motion) [9, 10]. The model we focus on in our recent research is the totally asymmetric simple exclusion process (TASEP), which can also be seen as a growth model (see Figure 1). Particles are on $\mathbb{Z}$ and they jump to their right with some given rate, provided the site is empty.

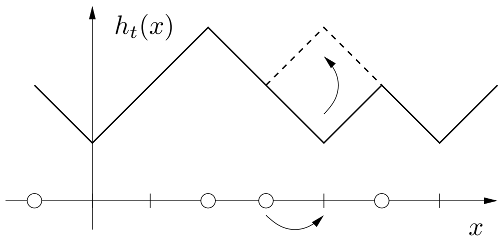

FIGURE 1. The growth model associated to the TASEP. The height function $h_t(x)$ increases by one unit if site x is empty and decreases by one unit if site x is occupied by a particle. Thus, when a particle jumps to the right, the the surface growth vertically as indicated by the dashed line.

A question: The TASEP belongs to the Kardar-Parisi-Zhang universality class [11]: the fluctuation of the position of a particle grows in time like $t^{1/3}$, while particles are correlated over distances of order $t^{2/3}$. What about the limit laws of fluctuations and limit process as time t becomes large?

The answer depends on the type of initial conditions (but not too much), i.e., from the point of view of fluctuations, the universality class has still to be divided into a few subclasses. Below are results known starting from deterministic initial configurations. Any bounded (t-independent) fluctuations in the initial conditions discussed below leads to the same limit result by a standard coupling argument.

Step initial conditions: In the TASEP, the *Airy$_2$ process*, $\mathcal{A}_2$, occurs from step initial conditions, where particles initially occupy $\mathbb{Z}_-$ only [9]. The macroscopic density of particles has a (linearly) *decreasing region*, in which the fluctuations of particle positions ($\sim$ height function h_t) are governed by the Airy$_2$ process in the large time limit. In this case, from the growth point of view, the *limit shape is curved*.

Periodic initial conditions: The analogue of the Airy$_2$ process in the case of *flat limit shape*, has been and called *Airy$_1$ process*, $\mathcal{A}_1$ [14, 3]. This occurs in the TASEP starting with periodic initial conditions, e.g., one particle every $d \geq 2$ sites of $\mathbb{Z}$ [3, 2]. Recently we also were able to prove that this new process describes the large time fluctuations in the PNG model, as expected by universality [4]. A review on the Airy$_2$ and Airy$_1$ processes including their known properties is [7].

Half-periodic initial conditions: However, in a typical situation one can have coexistence of the two processes in separated regions, joined by a transition process, which takes place over distances of order $t^{2/3}$. In that case, there is a transition process, between the Airy$_2$ and the Airy$_1$ process: the *Airy$_{2\to1}$ process*, $\mathcal{A}_{2\to1}$. This was discovered and described in [5], where particles start from $2\mathbb{Z}_-$, see Figure 2.

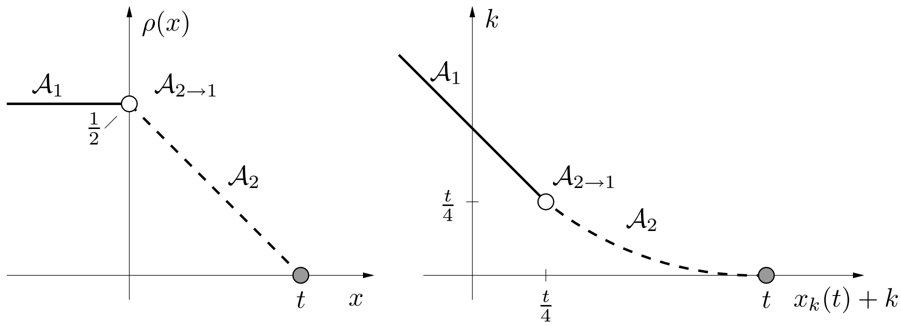

FIGURE 2. Left: the density ρ for large time t is linearly decreasing from $(0, 1/2)$ to $(t, 0)$. Right: The limit shape in an associated growth, obtained from the density ($k \in \mathbb{N}$ is the label of the particle which starts at $-2k$ and $x_k(t)$ is its position at time t).

Remark on stationary initial conditions: The stationary and translation invariant initial conditions are the Bernoulli-ρ measures on $\mathbb{Z}$, with $\rho \in [0, 1]$. Interesting fluctuations occurs along the characteristics, given by $j = (1 - 2\rho)t$. There the fluctuations lives on $t^{1/3}$ scale. The asymptotic distribution of the current as seen from an observer sitting on the characteristics has been analyzed [13, 8]. For a slower or faster observer, what it is seen are essentially only the fluctuations of the initial distribution, i.e., Gaussian on a $t^{1/2}$ scale.

Space-like paths extensions: In the PNG model, it was shown in [6] that the Airy_2 process occurs not only for fixed time, but for any space-like path (a path $t = \pi(x)$ is space-like if $|\pi'(x)| \leq 1$). This is the case also for the TASEP, but instead of *space and time* we have *time and particle number*, see [1] for details. The boundary cases of space-like paths for TASEP are: (1) fixed time and (2) fixed particle number (tagged particle). For step initial conditions we prove convergence to the Airy_1 process [1, 4]. For step initial conditions the same kind of result holds but with the Airy_2 as limit process.

References

[1] A. Borodin and P.L. Ferrari, *Large time asymptotics of growth models on space-like paths I: PushASEP*, preprint: arXiv:0707.2813 (2007).

[2] A. Borodin, P.L. Ferrari, and M. Prähofer, *Fluctuations in the discrete TASEP with periodic initial configurations and the Airy$_1$ process*, Int. Math. Res. Papers **2007** (2007), rpm002.

[3] A. Borodin, P.L. Ferrari, M. Prähofer, and T. Sasamoto, *Fluctuation properties of the TASEP with periodic initial configuration*, preprint: arXiv:math-ph/0608056; To appear in J. Stat. Phys. (2006).

[4] A. Borodin, P.L. Ferrari, and T. Sasamoto, *Large time asymptotics of growth models on space-like paths II: PNG and parallel TASEP*, preprint: arXiv:0707.4207 (2007).

[5] A. Borodin, P.L. Ferrari, and T. Sasamoto, *Transition between Airy$_1$ and Airy$_2$ processes and TASEP fluctuations*, preprint: arXiv:math-ph/0703023; To appear in Comm. Pure Appl. Math. (2007).

[6] A. Borodin and G. Olshanski, *Stochastic dynamics related to Plancherel measure*, AMS Transl.: Representation Theory, Dynamical Systems, and Asymptotic Combinatorics (V. Kaimanovich and A. Lodkin, eds.), 2006, pp. 9–22.

[7] P.L. Ferrari, *The universal Airy$_1$ and Airy$_2$ processes in the Totally Asymmetric Simple Exclusion Process*, preprint: arXiv:math-ph/0701021 (2007).

[8] P.L. Ferrari and H. Spohn, *Scaling limit for the space-time covariance of the stationary totally asymmetric simple exclusion process*, Comm. Math. Phys. **265** (2006), 1–44.

[9] K. Johansson, *Discrete polynuclear growth and determinantal processes*, Comm. Math. Phys. **242** (2003), 277–329.

[10] K. Johansson, *The arctic circle boundary and the Airy process*, Ann. Probab. **33** (2005), 1–30.

[11] K. Kardar, G. Parisi, and Y.Z. Zhang, *Dynamic scaling of growing interfaces*, Phys. Rev. Lett. **56** (1986), 889–892.

[12] M. Prähofer and H. Spohn, *Scale invariance of the PNG droplet and the Airy process*, J. Stat. Phys. **108** (2002), 1071–1106.

[13] M. Prähofer and H. Spohn, *Exact scaling function for one-dimensional stationary KPZ growth*, J. Stat. Phys. **115** (2004), 255–279.

[14] T. Sasamoto, *Spatial correlations of the 1D KPZ surface on a flat substrate*, J. Phys. A **38** (2005), L549–L556.

Reporter: