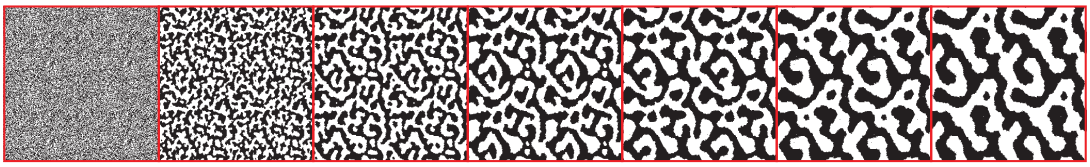
Master Seminar: Phase separation and interface evolution

WS 2014/2015

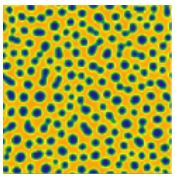
Prof. B. Niethammer, Dr. A. Schlichting (schlichting@iam.uni-bonn.de)



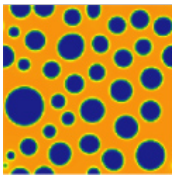
Phase separation during the Cahn-Hilliard evolution

In this seminar we will apply many different mathematical tools and concepts like calculus of variations, analysis for ODEs, existence for PDEs and asymptotic expansions to the problem of *phase separation*. Phase separation is an ubiquitous process, in which two components of a material inside a domain form pure phases with interfaces in between.

The driving mechanism is a competition between interface and bulk energies. The interface part of the energy penalizes transitions between the phases, whereas the bulk part prefers pure phases. The prototype of such energies is the *Ginzburg-Landau* energy, for which we investigate the associated minimization problem [M87].



The next step is to investigate the PDE leading to phase separation processes, which is the *Cahn-Hilliard* equation. The PDE decreases the energy in time and can be thought of a dynamic minimization of the Ginzburg-Landau energy. Besides clarifying the existence and uniqueness for this phase field model [EG96], we will also investigate the dynamics in the one-dimensional case [BH92].



In the next part, we establish the so called sharp interface limit [P89]. The limit, called *Mullins-Sekerka* equation, is a curvature driven interface evolution. The final aim is to deduce a simplified model for the coarsening behavior of the droplets during the Mullins-Sekerka evolution [N04].

Prerequisites: Introduction to PDEs, functional analysis

First meeting: Wednesday, 9th July, 12:15 in room 2.040

[BH92] L. Bronsard, D. Hilhorst, *On the Slow Dynamics for the Cahn-Hilliard Equation in one Space Dimension*, Proc. R. Soc. A: Math. Phys. Eng. Sci. 439 (1992) 669–682.

[EG96] C.M. Elliott, H. Garcke, *On the Cahn–Hilliard Equation with Degenerate Mobility*, SIAM J. Math. Anal. 27 (1996) 404–423.

[M87] L. Modica, *The gradient theory of phase transitions and the minimal interface criterion*, Arch. Ration. Mech. Anal. 98 (1987).

[N04] B. Niethammer, *Averaging methods for phase transition problems*, Lecture Notes, Rome, 2004.

[P89] R.L. Pego, Front Migration in the Nonlinear Cahn-Hilliard Equation, Proc. R. Soc. A: Math. Phys. Eng. Sci. 422 (1989) 261–278.