Coagulation-fragmentation equations

Graduate Seminar on Partial Differential Equations (S4B2)

Prof. Dr. Barbara Niethammer*, Iulia Cristian†

Summer Semester 2022

Description

Coagulation-fragmentation processes are of fundamental importance in a vast variety of applications such as aerosol physics, polymerization, blood agglomeration or social grouping.

They can be described by an evolution equation for the size distrubtion f of clusters of size ν. In the case of pure coaguation for example the basic model is:

$$\partial_t \overset{\#\text{ particles}}{\overset{\uparrow}{f(v,t)}} = \underbrace{\frac{1}{2}\int_{(0,v)} K(v-v',v')f(v',t)f(v-v',t)\mathrm{d}v'}_{\substack{\text{gain mass} \\ \uparrow \\ (\text{v-v'})+(\text{v'})=(\text{v})}} - \underbrace{\int_{(0,\infty)} \overbrace{K(v,v')}^{\text{collision rate}} f(v,t)f(v',t)\mathrm{d}v'}_{\substack{\text{lose mass} \\ \uparrow \\ \text{collision with other particles}}}$$

For pure coagualation processes, the so-called scaling hypothesis suggests that the long-time behaviour is described by self-similar solutions. This leads to the ansatz:

$$f(v,t) = \frac{1}{\sigma(t)^\tau} g(\frac{v}{\sigma(t)}),$$

where g solves the equation:

$$0 = \tau g(v) + v\partial_v g(v) + w\mathbb{K}[g(v)],$$

for some $w > 0$ and in which we denoted the coagulation term by $\mathbb{K}[g(v)]$.

If the break-up of clusters is also taken into account (in cases such as depolymerization and droplet break-up), additional fragmentation terms are added. Coagulation-fragmentation equations are considered to give a more realistic description for particle interactions, but both coagulation and fragmentation have applications when studied individually in special conditions.

In this seminar we will study the well-posedness of coagulation-fragmentation equations and the existence of self-similar solutions and their properties. Another relevant topic is the case when particles are injected into the system, which gives rise to an additional source term. We will also discuss recent results on the existence of steady states with source term.

*niethammer@iam.uni-bonn.de

†cristian@iam.uni-bonn.de

Prerequisites

Good knowledge of functional analysis.

Time

The seminar will take place

Monday, 16:15
Room 2.040

If you would like to attend the seminar, please send an e-mail to cristian@iam.uni-bonn.de before the start of the summer semester.

Possible Topics

- Introductory talk: methods used in coagulation-fragmentation equations (will not be assigned to students)
- **Existence theory**
  1. An existence proof for coagulation equations, see [SM89]
  2. An existence proof for coagulation-fragmentation equations, kernel with linear growth and integrable daughter distribution function, see [BLL20, Sections 8.2.2.1, 8.2.2.2]
  3. Improvements for the existence proof: non-integrable daughter distribution function, singular coagulation kernel, see [BLL20, Sections 8.2.2.3, 8.2.4]
- **Self-similar solutions**
  1. Proof of existence for self-similar solutions via discretization, see [BLL20, Section 10.2.4, p. 565-583]
  2. Regularity for self-similar solutions for coagulation equations, see [EM06, p. 321-351]
  3. Example where asymptotics can be derived for self-similar solutions of coagulation equations, see [NV12, without Solvability of the dual problem and Comparison argument]
- **Stationary solutions for equations with source term**
  1. Stationary solutions for coagulation equations when a source term is added in the L^1 context, see [Lau20]
  2. Stationary solutions for coagulation equations when a source term is added in the Radon measures context, part I, see [FLNV21, p. 809-836]
  3. Stationary solutions for coagulation equations when a source term is added in the Radon measures context, part II, see [FLNV21, finish Existence results: continuous model + Estimates and regularity]

References

[BLL20] J. Banasiak, W. Lamb, and P. Laurençot. *Analytic Methods for Coagulation-Fragmentation Models, Volume II.* CRC Press, 2020.

[EM06] M. Escobedo and S. Mischler. Dust and self-similarity for the Smoluchowski coagulation equation. *Annales de l'Institut Henri Poincaré C, Analyse non linéaire*, 23(3):331–362, 2006.

[FLNV21] M. Ferreira, J. Lukkarinen, A. Nota, and J. Velázquez. Stationary Non-equilibrium Solutions for Coagulation Systems. *Archive for Rational Mechanics and Analysis*, 240:809–875, 2021.

[Lau20] P. Laurençot. Stationary solutions to Smoluchowski's coagulation equation with source. *North-Western European Journal of Mathematics*, 6:137–164, 2020.

[NV12] B. Niethammer and J.J.L. Velázquez. Self-similar solutions with fat tails for Smoluchowski's coagulation equation with locally bounded kernels. *Communications in Mathematical Physics*, 318(2):505–532, 2012.

[SM89] I. W. Stewart and E. Meister. A global existence theorem for the general coagulation-fragmentation equation with unbounded kernels. *Mathematical Methods in the Applied Sciences*, 11(5):627–648, 1989.