S4B2 Graduate Seminar on PDE: Homogenization of Periodic Materials

Summer Semester 2024

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The behaviour of many microscopically heterogeneous materials, for example materials containing many tiny holes or very fine composite materials, can be approximated by suitable homogeneous materials with similar macroscopic properties. Mathematically, this means that the solution of a particular partial differential equation depending on a small parameter $\varepsilon > 0$ converges to the solution of a limit problem as $\varepsilon \to 0$. The goal of homogenization theory is to determine explicitly the limit problem and to prove the convergence of solutions in an appropriate space.

We begin with the homogenization of PDEs with rapidly oscillating coefficients arising from periodic composite materials. We cover both Tartar's proof method of oscillating test functions and the two-scale convergence method. Then, we discuss materials with periodic holes and examine the different limit problems for Dirichlet and Neumann boundary values.

Prerequisites: basic PDE, basic functional analysis

Timeslot: Mondays at 16:15 in room 2.040

Preliminary meeting: Monday, January 29th at 16:15 in room 2.040

References

[1] Grégoire Allaire. Homogenization and two-scale convergence. *SIAM Journal on Mathematical Analysis*, 23(6):1482–1518.

[2] Doina Cioranescu and Patrizia Donato. *An Introduction to Homogenization*. Oxford University Press.

[3] Doina Cioranescu and François Murat. A strange term coming from nowhere. In Andrej V. Cherkaev and Robert Kohn, editors, *Topics in the Mathematical Modelling of Composite Materials*, pages 45–93. Springer International Publishing.

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