Advanced topics in Analysis and PDE (V5B1)

Analysis of coagulation-fragmentation equations

WT 2021/22

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Description:

Coagulation equations are fundamental models to describe mass aggregation through coalescence of clusters. Traditionally the largest application area is physical chemistry, where coalescence processes occur in polymerization, formation of aerosols or in liquid mixtures, but aggregation processes are also relevant in various other branches of science such as astronomy and mathematical biology.

The basic mathematical model goes back to Smoluchowski and describes the evolution of the clusters through the number density $f(t,x)$ of clusters of size x. Clusters of size x and y can coalesce by binary collisions to clusters of size $x+y$ at a rate given by a rate kernel $K(x,y)$, such that the dynamics of f are given by

$$\frac{\partial}{\partial t} f(t,x) = \frac{1}{2}\int_0^x K(y,x-y) f(t,x-y) f(t,y)\, dy \;-\; f(t,x)\int_0^\infty K(x,y) f(t,y)\, dy\,, \tag{1}$$

supplemented with an initial condition at $t = 0$. The qualitative behaviour of solutions to (1) depends crucially on the form of the rate kernel $K(x,y)$ in which all the details of the specific coagulation process are subsumed.

In this lecture we will discuss the well-posedness theory of (1), the phenomenon of gelation, i.e. the loss of mass in finite time, as well as the long-time behaviour of solutions.

We will also discuss aspects of so-called coagulation-fragmentation equations where also the breakage of clusters into smaller ones is taken into account.

A recent book on coagulation-fragmentation equations is listed below. In the lecture we will also discuss originial research articles.

Prerequisites:
Good knowldege in Integration Theory and Functional Analysis

Time:

Tuesday and Wednesday 10-12h
Room 2.040

Literature:

J. Banasiak, W. Lamb and P. Laurençot: Analytic Methods for Coagulation-Fragmentation MOdels, Vol. I+II, CRC Press, 2020